

Lesson 11.6

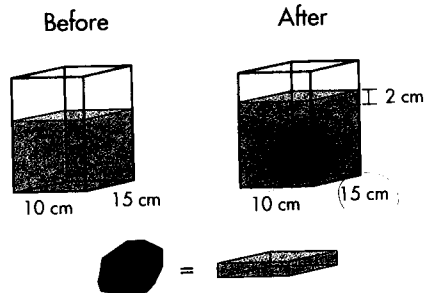
Displacement and Density

What will happen if you step into a bathtub that is filled to the brim? Right. The water will overflow. What will happen if you fill a glass to the brim with root beer and then add a scoop of ice cream? Right. You'll have a mess! The volume that overflows in each of these situations equals the volume of the solid below the liquid level. This volume is called an object's **displacement**. You can determine the volume of an irregularly shaped object by measuring its displacement.

Example A



Geologist Crystal Stone wishes to calculate the volume of an irregularly shaped rock. She places it into a rectangular prism containing water. The base of the container measures 10 cm by 15 cm. When the rock is put into the container, Crystal notices that the water level rises 2 cm because the rock displaces its volume of water. This new "slice" of water has a volume of $(2)(10)(15)$, or 300 cm^3 . Therefore the volume of the rock is 300 cm^3 .



An important property of a material is its density. **Density** is the mass of matter in a given volume. Density is calculated by dividing the mass in grams by the volume in cubic centimeters (density = $\frac{\text{mass}}{\text{volume}}$). A chemist wishing to identify an unknown clump of metal could weigh the clump to determine its mass, determine its volume by displacement, calculate its density, and, finally, look in a chemical handbook in hopes of identifying the compound by its density.

Metal	Density	Metal	Density
Aluminum	2.81 g/cm ³	Nickel	8.89 g/cm ³
Copper	8.97 g/cm ³	Platinum	21.40 g/cm ³
Gold	19.30 g/cm ³	Potassium	0.86 g/cm ³
Lead	11.30 g/cm ³	Silver	10.50 g/cm ³
Lithium	0.54 g/cm ³	Sodium	0.97 g/cm ³

Example B

A clump of metal weighing 351.4 g is dropped into a cylindrical container, causing the water level to rise 1.1 cm. The radius of the base of the container is 3.0 cm. What is the density of the metal? Assuming the metal is pure, what is the metal?

$$\begin{aligned} \text{Volume} &= \pi(3.0)^2(1.1) \\ &= (\pi)(9)(1.1) \\ &\approx 31.1 \end{aligned}$$

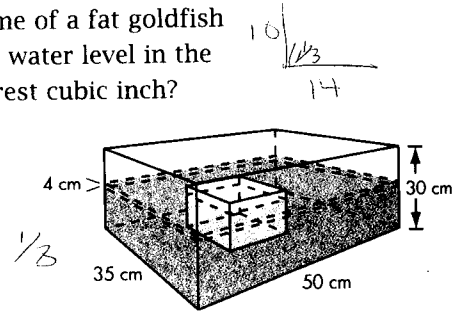
$$\begin{aligned} \text{Density} &\approx \frac{351.4}{31.1} \\ &\approx 11.3 \end{aligned}$$

The density is 11.3 g/cm^3 . Therefore the metal is lead.

Exercise Set 11.6

1. A rock is added to a container of water, and it raises the water level 3 cm. If the container is a rectangular prism whose base measures 15 cm by 15 cm, what is the volume of the rock?
2. A solid glass ball is dropped into a cylinder with a radius of 6 cm, raising the water level 1 cm. What is the volume of the glass ball?
3. A fish tank 10 inches by 14 inches by 12 inches high is the home of a fat goldfish named Columbus. When he is taken out for some fresh air, the water level in the tank drops $\frac{1}{3}$ inch. What is the volume of Columbus to the nearest cubic inch?

- 4.* A block of ice is placed into an ice chest containing water. The ice causes the water level to rise 4 cm. The ice chest measures 35 cm by 50 cm by 30 cm high. When ice floats in water, one eighth of its volume floats above the water level and seven eighths floats beneath the water level. What is the volume of the block of ice?

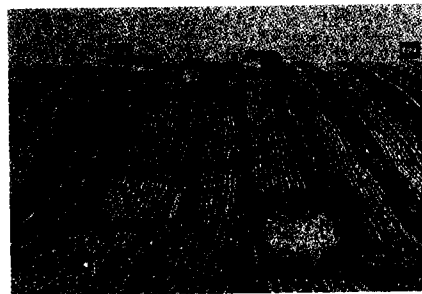


5. A piece of wood placed in a cylindrical container causes the container's water level to rise 3 cm. This type of wood floats half out of the water, and the radius of the container is 5 cm. What is the volume of the piece of wood?

6. Sandy Pyle has found a clump of metal, that appears to be either silver or lead. Hoping it is silver, she drops it into a rectangular container filled halfway with water. She observes that the water level rises 2 cm. Because the base of the container is a square measuring 5 cm on an edge, she is able to calculate the volume of the clump. She weighs the metal and finds that it weighs 525 g. What is the density of the metal? Is it lead or silver?

- 7.* Chemist Donna Dalton is given a clump of metal and is told that it is sodium. She weighs the metal and finds that it weighs 145.5 grams. To test if it is indeed sodium, she places it into a square prism whose base measures 10 cm on each edge and that is partially filled with a nonreactive liquid. If the metal is sodium, how many centimeters will the liquid level rise?

8. The round bales shown at right are 4 feet in diameter and 4 feet long. Calculate the volume of each bale. Bales this size weigh approximately 900 lbs. What is the density of the packed hay? Square balers make bales whose dimensions are $4' \times 4' \times 6'$ and that weigh approximately 1400 lbs. Is this hay packed more or less densely than the hay in a round bale? Explain.



9. How much does a solid block of aluminum weigh if its dimensions are 4 cm by 8 cm by 20 cm?

10. Which will weigh more: a solid cylinder of gold with a height of 5 cm and a diameter of 6 cm or a solid cone of platinum with a height of 21 cm and a diameter of 8 cm? Which is worth more?



16 $\frac{900}{1400}$

$\frac{5}{19}$

$\frac{525}{50}$

$\frac{145.5}{\times 150}$

11. Each edge of this alunite crystal pyramid measures 6 cm. You want to drop it into a shallow cylindrical container of crystal-cleansing fluid. If the diameter of the container is 10 cm and the height is 5 cm, will the crystal fit? How much cleansing fluid should you put into the container to cover the crystal without letting the fluid overflow? Explain your reasoning.



12. Sherlock Holmes has just returned home excited. He rushes to his chemical lab, takes a mysterious medallion out of his carrying case, and weighs it. "It weighs 3088 grams," Mr. Holmes says in anticipation. "Now, let's check its volume." He pours water into a glass graduated container with a square base that measures 10 centimeters on each side, then records the water level, which is 53.0 centimeters. He places the medallion into the container and reads the new water level, 54.6 centimeters. After a few minutes of the mental calculation that Holmes enjoys so much, he turns to Dr. Watson. "This confirms my theory about Colonel Banderson. Quick, Watson! The game is afoot. We are off to the train station."

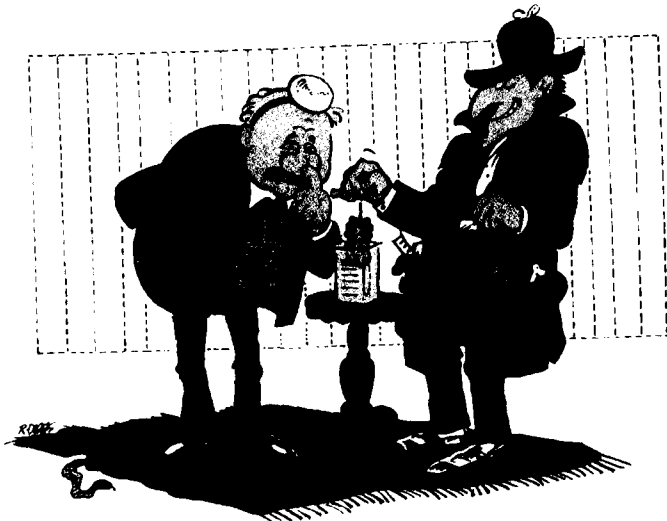
Poor Dr. Watson is still standing with the *London Daily* in his hands. "Holmes, you amaze me. Is it gold?" questions the good doctor.

"If it has a density of 19.3 grams per cubic centimeter, it is gold," smiles Mr. Holmes. "If it is gold, then the Colonel is who he says he is. If it is a fake, then so is Colonel Banderson."

Watson is still waiting for the answer. "Well?" he queries.

Holmes, heading for the door, smiles and says, "It's elementary, my dear Watson. Elementary geometry, that is."

What is the volume of the medallion? Is it gold and is Colonel Banderson who he says he is?



Sherlock Holmes and the mysterious medallion



Improving Algebra Skills—An Algebra Mind-Reading Trick II

Try the following card trick. Use algebra to explain why the trick works.

As the mathemagician, ask your friend to give you any two cards whose sum is less than 10. Tell your friend you will be able to pick a third card that you can use with the other two to make a three-digit number that is divisible by 11. The trick is to add them and find a card equal to that sum.