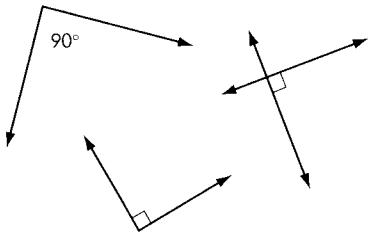
Investigation 2.3.2

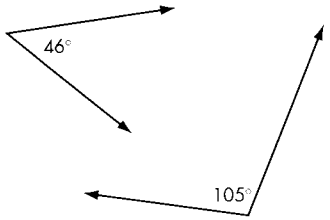
Use the information provided to write a good definition of each geometric figure below. Once you are satisfied with the definitions you have written, discuss them with others near you. See if your classmates can find counterexamples that will reveal problems with your definitions. If they do, refine your definitions until you are satisfied with them. Compare your refined definitions with those of others. Try to arrive at common definitions that your class can agree on. Add these common definitions to the definition list you have started in your notebook. Draw and label a picture to illustrate each definition. You and your classmates are responsible for the definitions in this course—there is no glossary of terms in this text because defining terms yourself will deepen your understanding and help you remember them better. Therefore, it is very important that you keep an accurate and updated notebook listing of the class-accepted definitions.

1.* Define *right angle.*

Right angles



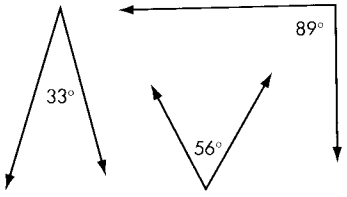
Not right angles



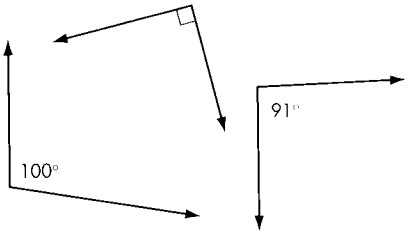
Note: A small square in the corner of an angle indicates that it measures 90°.

2.* Define *acute angle.*

Acute angles

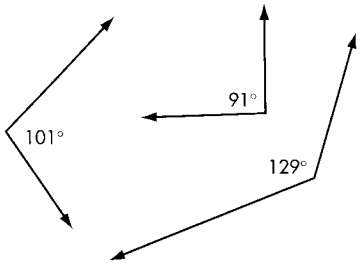


Not acute angles

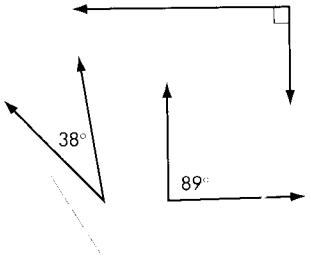


3. Define *obtuse angle.*

Obtuse angles

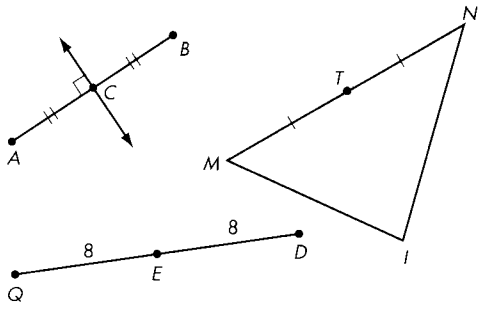


Not obtuse angles



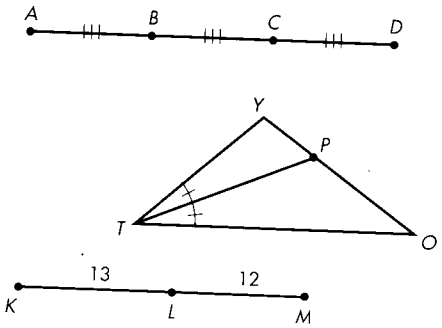
4. Define *midpoint of a segment.*

Midpoints of segments



Point C is a midpoint of segment AB.
Point T is a midpoint of segment MN.
Point E is a midpoint of segment QD.

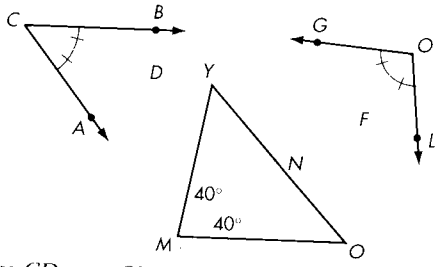
Not midpoints of segments



Points B and C are not midpoints of segment AD.
Point P is not a midpoint of segment OY.
Point L is not a midpoint of segment KM.

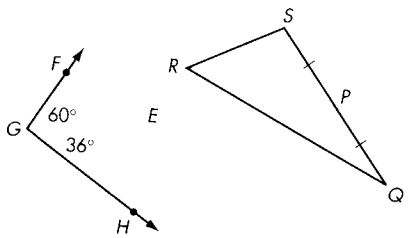
5. Define *angle bisector.*

Angle bisectors



Ray CD, ray OF, and ray MN are angle bisectors.

Not angle bisectors

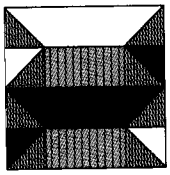


Ray GE and ray RP are not angle bisectors.

Exercise Set 2.3

In Exercises 1–5, draw and carefully label the figures. Use the appropriate marks to indicate right angles and to indicate segments and angles equal in measure. Use a protractor and a ruler when necessary.

1. Acute angle DOG with a measure of 45 degrees
2. Right angle RTE
3. Obtuse angle BIG with angle bisector $\overrightarrow{IE}$
4. Line segment OY with midpoint L
5. Show how three lines in a plane can intersect in no points, exactly one point, exactly two points, or exactly three points.



Lesson 2.4

Defining Line and Angle Relationships

There are two kinds of people in this world: those who divide everything into two groups, and those who don't.

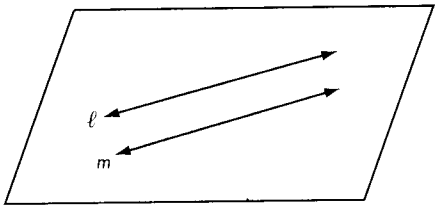
— Kenneth Boulding

Investigation 2.4

Write a good definition for each geometric term or figure below. Once you are satisfied with your definitions, discuss them with others. Try to arrive at one common definition your class can agree on for each term. Add these definitions to the definition list in your notebook. Draw and label a picture to illustrate each definition.

1. Define *parallel lines.*

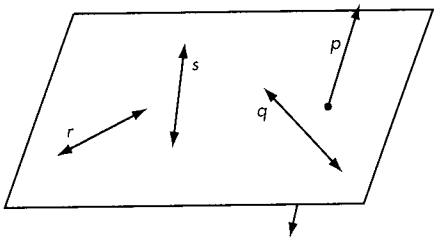
Parallel lines



$\ell \parallel m$

Note: Lines are sometimes labeled and named with lowercase letters. The symbol $\parallel$ means "is parallel to."

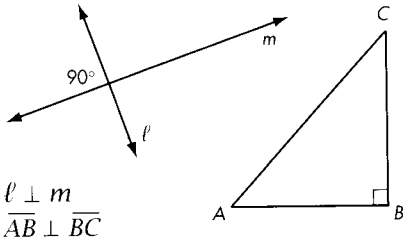
Not parallel lines



Line r is not parallel to line s.
Line p is not parallel to line q.
Note: Lines p and q are not in the same plane. Such lines are called **skew** lines.

2. Define *perpendicular lines.*

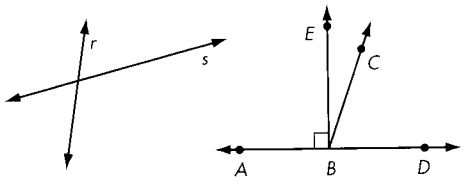
Perpendicular lines



$\ell \perp m$
$\overline{AB} \perp \overline{BC}$

Note: The symbol $\perp$ means "is perpendicular to."

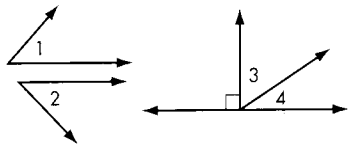
Not perpendicular lines



Line r is not perpendicular to line s.
Ray BC is not perpendicular to line AD.

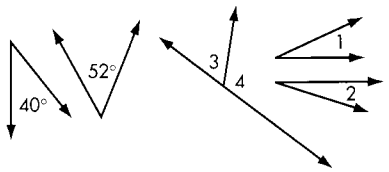
3. Define *pair of complementary angles.*

Pairs of complementary angles



$m\angle 1 + m\angle 2 = 90°$
$m\angle 3 + m\angle 4 = 90°$

Not pairs of complementary angles

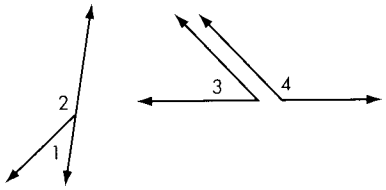


$m\angle 1 + m\angle 2 < 90°$

Note: Sometimes it's convenient to name angles in a diagram with a number.

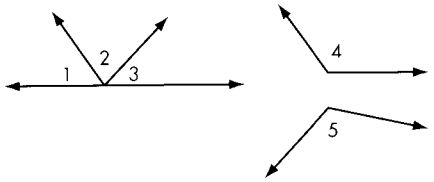
4. Define *pair of supplementary angles.*

Pairs of supplementary angles



$m\angle 1 + m\angle 2 = 180°$
$m\angle 3 + m\angle 4 = 180°$

Not pairs of supplementary angles



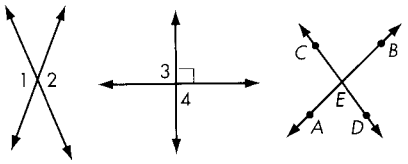
$m\angle 1 + m\angle 2 < 180°$
$m\angle 4 + m\angle 5 > 180°$

Quite often geometric definitions are easier to write if you refer to named figures. For example, you can define a line segment by saying: "Segment AB consists of points A and B and all points on $\overleftrightarrow{AB}$ that are between points A and B." You can define a ray by saying: "Ray AB consists of $\overline{AB}$ and all other points P on $\overleftrightarrow{AB}$ such that point B is between points A and P." You can define the midpoint of a line segment by saying: "Point M is a midpoint of $\overline{AB}$ if M is a point on $\overline{AB}$ and AM equals MB."

The definitions for the next two terms are easier to write if you use this technique.

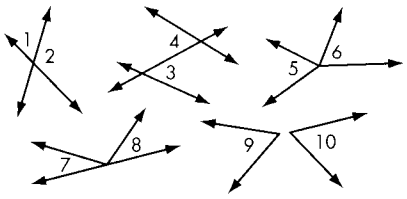
5.* Define *pair of vertical angles.*

Pairs of vertical angles



$\angle 1$ and $\angle 2$ are a pair of vertical angles.
$\angle 3$ and $\angle 4$ are also vertical angles.
$\angle AED$ and $\angle BEC$ are also vertical angles.

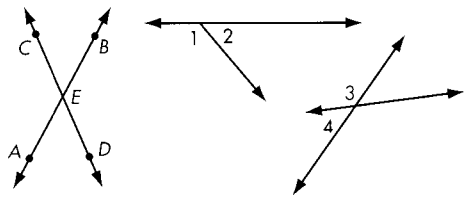
Not pairs of vertical angles



$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$, $\angle 7$ and $\angle 8$, and $\angle 9$ and $\angle 10$ are not pairs of vertical angles.

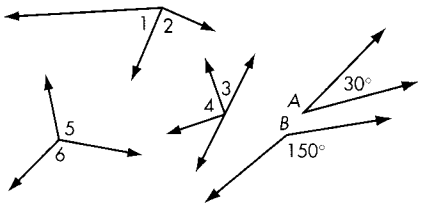
6.* Define *linear pair of angles.*

Linear pairs of angles



$\angle 1$ and $\angle 2$ are a linear pair of angles.
$\angle 3$ and $\angle 4$ are a linear pair of angles.
$\angle AED$ and $\angle AEC$ are a linear pair of angles.

Not linear pairs of angles



$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$, and $\angle A$ and $\angle B$ are not linear pairs of angles.

Exercise Set 2.4

This exercise set will help you visualize relationships between geometric figures in the plane and in space. In Exercises 1–10, all but three of the statements are true. Make a sketch or use physical objects to demonstrate each true statement. For the three false statements, produce a counterexample demonstrating that each is false. If you wish to create physical models, pencil tips and thumbtacks can represent points. Rulers, pencils, or stiff wires can represent lines.

1.* For every line segment there is exactly one midpoint.

2. For every angle there is exactly one angle bisector.

3. If two different lines intersect, then they intersect at one and only one point.

4. If two different circles intersect, then they intersect at one and only one point.

5.* There is one and only one line perpendicular to a given line through a given point on the given line.

6. In a plane there is exactly one line perpendicular to a given line through a given point on the given line.

7. There is exactly one line perpendicular to a given line through a given point not on the given line.

8. In every triangle there is exactly one right angle.

9. Through a given point not on a given line there is one and only one line that can be constructed parallel to the given line.

10. It is possible for two triangles to intersect in one point, two points, three points, four points, five points, or six points, but not exactly seven points.

In Exercises 11–14, draw and carefully label the figure described. Use a protractor and a ruler when necessary.

11. $\overline{PE}$ perpendicular to $\overline{AR}$

12. Vertical angles ABC and DBE

13. Complementary angles A and B with $m\angle A = 40°$

14. Supplementary angles C and D with $m\angle D = 40°$