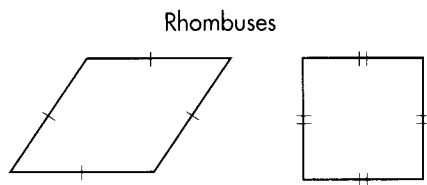
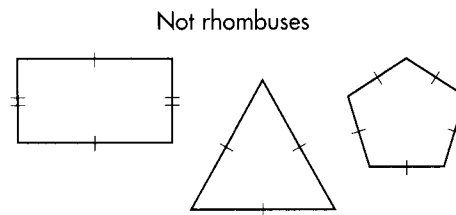




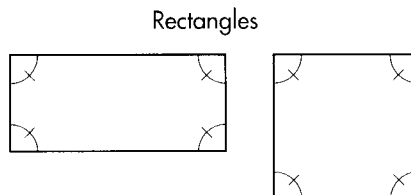
4. Define *rhombus*.



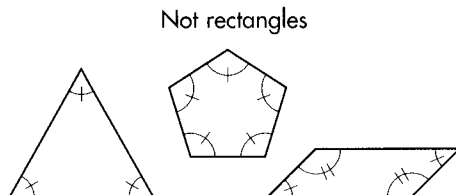
Both polygons are parallelograms.



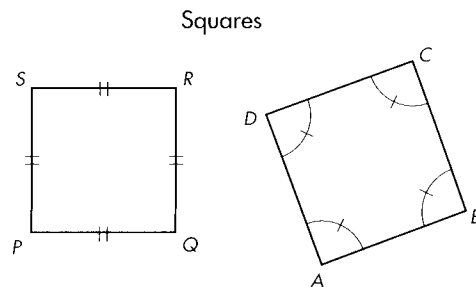
- 5.\* Define *rectangle*.



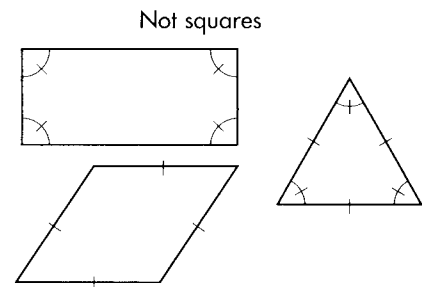
Both polygons are parallelograms.



- 6.\* Define *square*.



Quadrilateral  $ABCD$  is a rhombus.  
Quadrilateral  $PQRS$  is a rectangle.



## Exercise Set 2.7

In Exercises 1-10, write the words or the symbols that make the statement true.

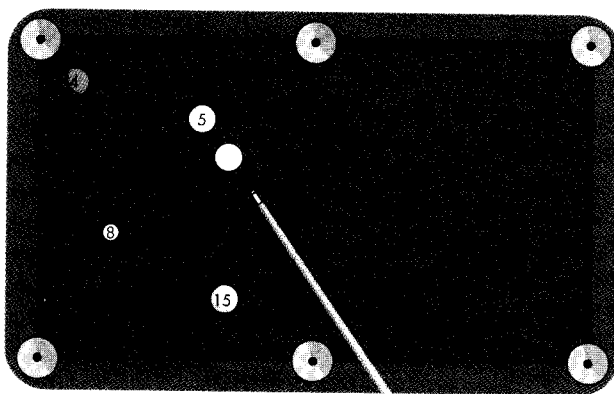
1. The three undefined elements of geometry are  $-\text{?}-$ ,  $-\text{?}-$ , and  $-\text{?}-$ .
2. "Line  $AB$ " is written in symbolic form as  $-\text{?}-$ .
3. "Line segment  $AB$ " is written in symbolic form as  $-\text{?}-$ .
4. "Ray  $AB$ " is written in symbolic form as  $-\text{?}-$ .
5. "Angle  $ABC$ " is written in symbolic form as  $-\text{?}-$ .
6. The point where the two rays of an angle meet is called the  $-\text{?}-$ .
7. The geometric tool used to measure an angle is called a  $-\text{?}-$ .
8. "Line  $AB$  is parallel to segment  $CD$ " is written in symbolic form as  $-\text{?}-$ .
9. The sentence "Segment  $AB$  is perpendicular to line  $CD$ " is written in symbolic form as  $-\text{?}-$ .
10. The angle formed by a light ray coming into a mirror is  $-\text{?}-$  the angle formed by a light ray leaving the mirror.

In Exercises 11–22, identify the statement as true or false. For each false statement, sketch a counterexample that proves the statement false.

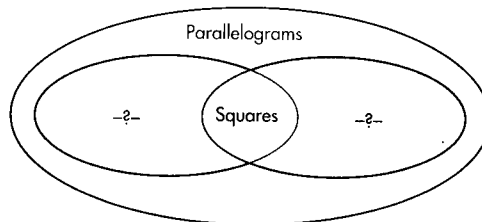
11. An angle is measured in degrees.
12. An acute angle is an angle whose measure is less than  $90^\circ$ .
13. An obtuse triangle has exactly one angle whose measure is greater than  $90^\circ$ .
14. A diagonal is a line segment that connects any two vertices of a polygon.
15. If the measures of two angles add up to  $90^\circ$ , then they are supplementary.
16. If two lines intersect to form a right angle, then the lines are perpendicular.
17. An angle bisector is any ray or segment that divides an angle into two angles.
18. A median of a triangle is a line segment from a vertex to the midpoint of the opposite side.
19. A trapezoid is a device used to capture zoids.
20. A rhombus is a parallelogram whose sides all have the same measure.
21. A kite is a parallelogram.
22. Every square is a rectangle.

In Exercises 23–28, draw a triangle that fits the name. If impossible, write “not possible.” Use your geometric tools to make your drawings as accurate as possible.

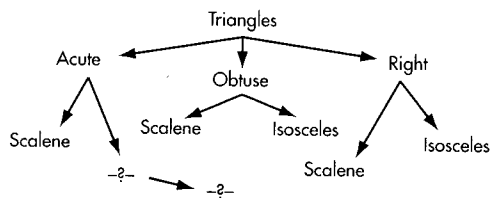
23. Scalene triangle
24. Scalene acute triangle
25. Obtuse isosceles triangle
26. Isosceles right triangle
27. Equilateral right triangle
28. Scalene isosceles triangle
- 29.\* Copy the pool table and the balls shown. If the cue ball is hit as indicated, show the path of the five ball as it hits three cushions before coming to rest.



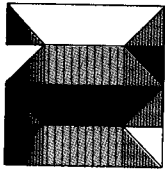
30. At right is a concept map showing the relationships between some members of the parallelogram family. This type of concept map is known as a **Venn diagram**. Fill in the missing names.
31. Many of the geometric figures you have defined are closely related to one another. A diagram can help you see the relationships between them. At right is a concept map showing the relationships between members of the triangle family. This type of concept map is known as a **tree diagram** because the relationships are shown as branches of a tree. Copy and fill in the missing branches of the tree diagram for triangles.
32. What types of quadrilaterals have one or more lines of symmetry? Describe the reflectional symmetry of these special quadrilaterals and write new symmetry-based definitions for them.



Venn diagram for parallelograms



Tree diagram for triangles



## Lesson 2.8

# Space Geometry

*Space: The final frontier*  
— Star Trek

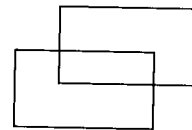
In the beginning lessons of this chapter you were introduced to *point*, *line*, and *plane*, and you used those terms to define a wide range of other geometric figures, from rays to polygons. All of your work, however, was done on a single flat surface, a single plane. In this lesson you will learn a little about space geometry: the geometry of objects that are not restricted to a flat surface (not necessarily coplanar). The geometric solid you are probably most familiar with is the basic box, or rectangular solid. You learned how to draw boxes using perspective in Chapter 0. Perspective makes solids look more realistic, so it's often used in art. But for the purpose of doing geometry, it's not usually necessary to draw solids in perspective. Below are tips for drawing rectangular solids without perspective.

Step 1

Step 2

Step 3

Step 4



Rectangular solid (face closest to the viewer)

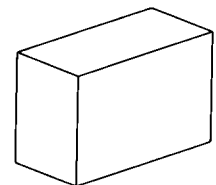
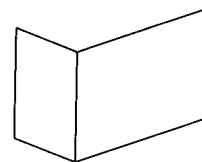
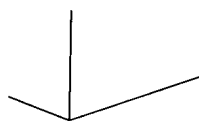
Use dashed lines to represent lines that would not be visible if this object were actually solid.

Step 1

Step 2

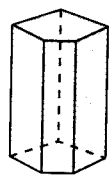
Step 3

Step 4

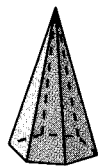


Rectangular solid (edge closest to the viewer)

Some other three-dimensional objects or space figures you will study include the six basic geometric solids shown below.



Prism



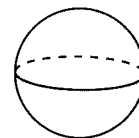
Pyramid



Cylinder



Cone



Sphere



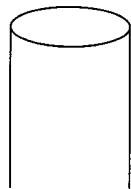
Hemisphere

Formal definitions of these solids will come in a later chapter. The shapes of the solids are probably already familiar to you even if you are not familiar with their formal names. The ability to draw these geometric solids is an important visual-thinking skill. Here are some drawing tips. Don't forget the hidden lines.

Cylinder

Step 1

Step 2



Cone

Step 1

Step 2



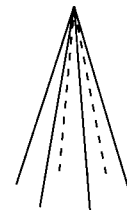
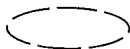
Pyramid

Step 1

Step 2

Step 3

Step 4



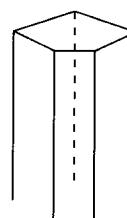
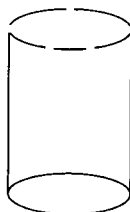
Prism

Step 1

Step 2

Step 3

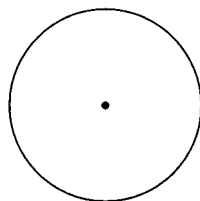
Step 4



Sphere

Step 1

Step 2



Hemisphere

Step 1

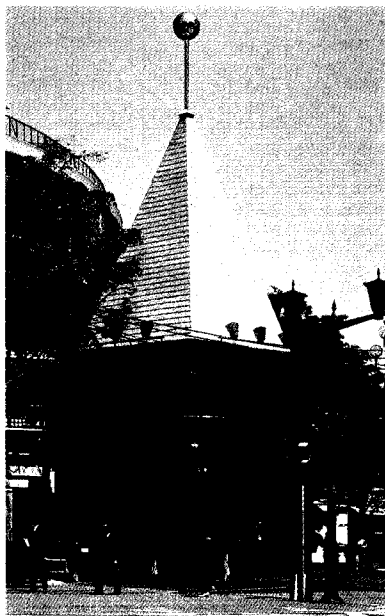
Step 2



## Exercise Set 2.8

In Exercises 1-6, draw the figure. Study the drawing tips provided on the previous page before you start.

1. Cylinder
2. Cone
3. Prism with a hexagonal base
4. Pyramid with a heptagonal base
5. Sphere
6. Hemisphere
7. This small neighborhood police station (called a *koban*) in Tokyo is a prism-shaped building with a pyramid roof and a cylindrical porch. Draw a cylindrical building with a cone roof and a prism-shaped porch.

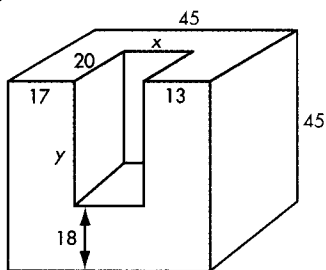


In Exercises 8-11, draw and label the solid figure described. In Exercises 8-10, make a scaled drawing of the figure. For example, in Exercise 9 you are asked to draw a rectangular solid 2 m by 3 m by 4 m. There is no way that you're going to draw a solid that is actually that big. Draw the solid at a smaller scale so that the dimensions are about 2 units by 3 units by 4 units, then label the figure with meters.

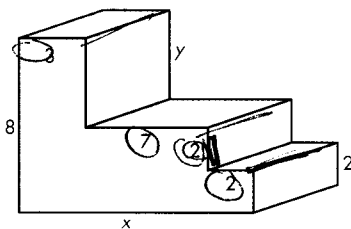
8. A rectangular solid with a base 6 cm by 12 cm, a height of 3 cm, and a face closest to the viewer
9. A rectangular solid 2 m by 3 m by 4 m, sitting on its biggest face, with an edge closest to the viewer
- 10.\* A rectangular solid 3 inches by 4 inches by 5 inches resting on its smallest face. Draw lines on the three visible surfaces showing how the solid can be divided into 60 cubic-inch boxes.
11. A cube (a prism, each face being a square). Draw lines on the three visible faces to show how it can be divided into 27 identical smaller cubes.

In Exercises 12-14, find the lengths  $x$  and  $y$ . (Each angle on each polygonal side of the block is a right angle.)

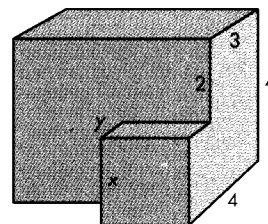
12.



13.



14.



Each figure in Exercises 15-17 represents a card with a wire attached. The three-dimensional figure formed by spinning the two-dimensional figure on the wire between your fingers is called a **solid of revolution**. Sketch the solid of revolution formed when the two-dimensional figure is revolved by spinning the wire.

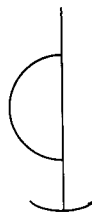
15.\*



16.

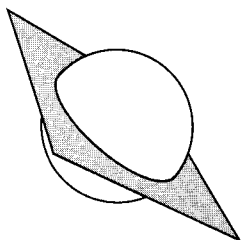


17.

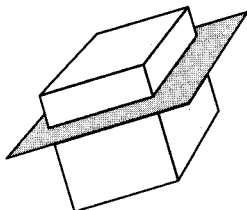


When a solid is cut by a plane, the resulting plane figure is called a **section**. In Exercises 18-21, sketch the section formed when the solid is sliced by the plane, as shown.

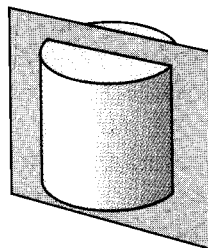
18.



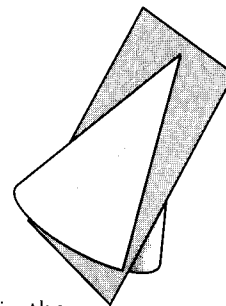
19.



20.

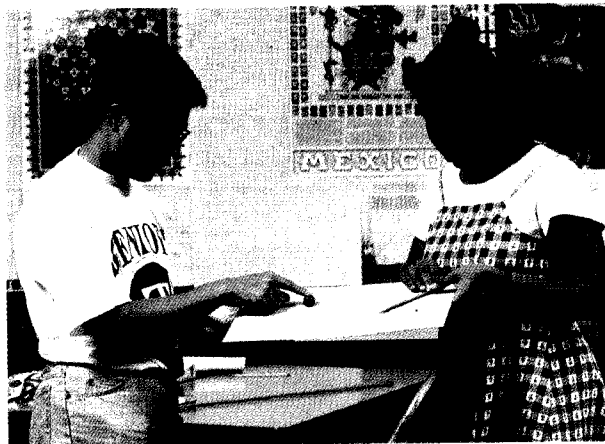


21.

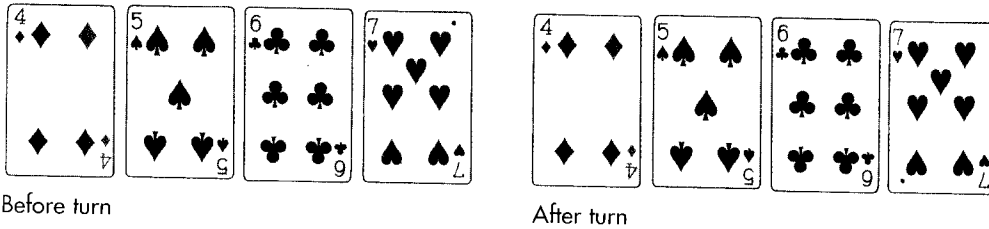


This set of exercises will help you visualize relationships between geometric figures in the plane and in space. All the statements in Exercises 22-34 are true except two. Make a sketch or use physical objects to demonstrate each true statement. For the two false statements, produce a counterexample demonstrating that each is false. Pencil tips and thumbtacks can represent points. Rulers, pencils, spaghetti, or stiff wires can represent lines. Sheets of heavy paper, cardboard, or your desk top can represent planes.

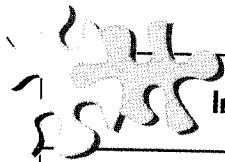
22. One and only one distinct line can be drawn through two different points.
23. One and only one distinct plane can be made to pass through three noncollinear points.
24. Exactly one distinct plane passes through one line and a point not on the line.
25. If a line intersects a plane not containing it, then the intersection is exactly one point.
- 26.\* If two lines are perpendicular to the same line, then they are parallel.
27. If two different planes intersect, then their intersection is a line.
28. If a line and a plane have no points in common, then they are parallel.
29. If two coplanar lines are both perpendicular to a third line in the same plane, then the two lines are parallel.
30. If two different planes do not intersect, then they are parallel.



31. If a plane intersects two parallel planes, then the lines of intersection are parallel.
32. If three random planes intersect (no two parallel and no three through the same line), then they divide space into six parts.
33. If a line is perpendicular to two lines in a plane, but the line is not contained in the plane, then the line is perpendicular to the plane.
- 34.\* If two lines are perpendicular to the same plane, then they are parallel to each other.
35. Miriam the Magician placed four cards faceup on her magic table (the four cards shown below left). Blindfolded, she instructed someone from her audience to come up to the stage and turn exactly one card around  $180^\circ$ .

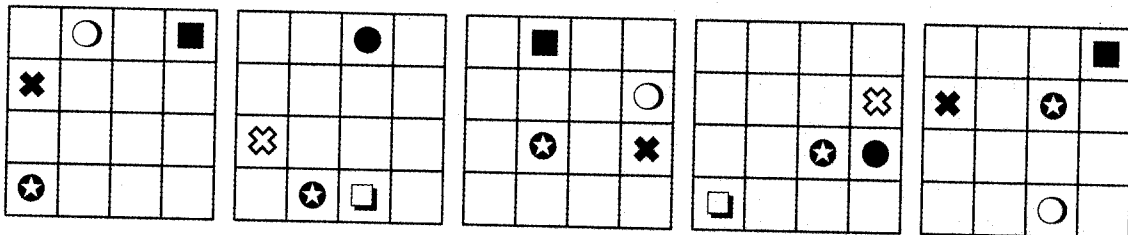


Miriam removed her blindfold and claimed she was able to determine which card was turned  $180^\circ$  by "magically reading the psychic energy emanating from the cards." Of course, it is possible to figure out which card was turned around without magic. Can you figure out which card was turned?



### Improving Visual Thinking Skills—Picture Patterns I

Here is a sequence you "read" left to right.



Which comes next?

