Lesson 7.2

Discovering Chord Properties

In this lesson you will discover some properties of chords, arcs, and central angles. Investigation 7.2.1 is about chords and central angles.

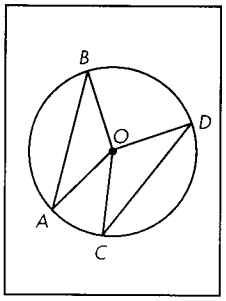
Investigation 7.2.1

Step 1 Construct a large circle. Label the center O.

Step 2 Construct two congruent chords in your circle. (Use your compass to guarantee that they are congruent.) Label the chords $\overline{AB}$ and $\overline{CD}$.

Step 3 Construct radii $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$.

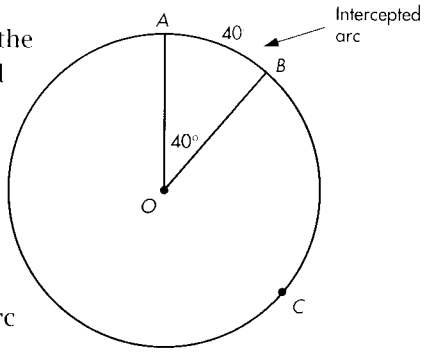
Step 4 With your protractor, measure $\angle BOA$ and $\angle COD$.



Compare your results with the results of others near you. State your observations as your next conjecture.

C-61 If two chords in a circle are congruent, then they determine two central angles that are —?—.

The measure of a minor arc is defined as the measure of its central angle. For example, the central angle $\angle BOA$ at right has a measure of 40° and, therefore, the measure of the intercepted arc $\overset{\frown}{AB}$ is 40° (written as $m\overset{\frown}{AB} = 40°$). A semicircle has a measure of 180°. A circle has a measure of 360°. The measure of a major arc is 360° minus the measure of the minor arc making up the remainder of the circle. For example, the measure of major arc $\overset{\frown}{BCA}$ is 360° - 40°, or 320°.



Your next conjecture follows almost immediately from Conjecture 61 and the definition of arc measure. Two congruent chords in a circle determine two central angles that are congruent. And it follows from the definition of arc measure that if two central angles are congruent, their intercepted arcs must be congruent. These two statements can be linked to show a relationship between chords and their arcs.

C-62 If two chords in a circle are congruent, then their —?— are congruent.

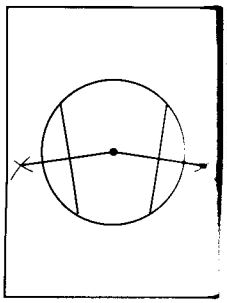
In Investigations 7.2.2 and 7.2.3, you will discover relationships about chords that are congruent and chords that are not congruent.

Investigation 7.2.2

Step 1 Construct a large circle. Mark the center.

Step 2 Construct two nonparallel congruent chords that are not diameters.

Step 3 Construct the perpendiculars from the center to each chord.



How does the perpendicular from the center of a circle to a chord divide the chord? State your observations as a conjecture.

C-63 The perpendicular from the center of a circle to a chord is the —?— of the chord.

Let's continue this investigation to discover a relationship between congruent chords and their distances from the center of the circle.

Step 4 With your compass, compare the distances (measured along the perpendicular) from the center to the chords.

State your observations as your next conjecture.

C-64 Two congruent chords in a circle are —?— from the center of the circle.

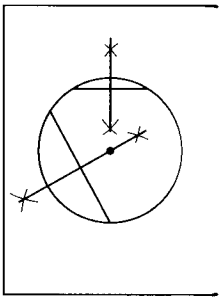
In Investigation 7.2.3, you will discover a property of perpendicular bisectors of chords in a circle.

Investigation 7.2.3

Step 1 Construct a large circle and mark the center.

Step 2 Construct two nonparallel chords that are not diameters.

Step 3 Construct the perpendicular bisector of each chord and extend the bisectors until they intersect.



What is special about the point of intersection? Compare your results with the results of others near you. State your observations as a conjecture. This is the converse of Conjecture 63.

C-65 The perpendicular bisector of a chord —?—.

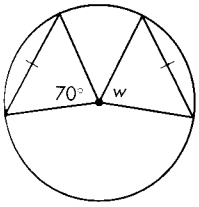
Take Another Look 7.2

1. Use a geometry computer program or patty papers to confirm one chord conjecture.
2. Explain how Conjecture 63 follows logically from a kite conjecture.
3.* Explain why the perpendicular bisector of a chord passes through the circle's center.

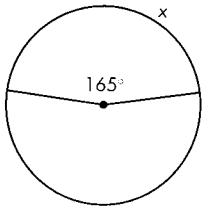
Exercise Set 7.2

Solve Exercises 1–6. State which conjecture or definition you used to support your conclusion.

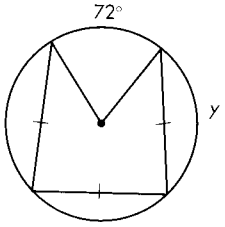
1. w = -?-



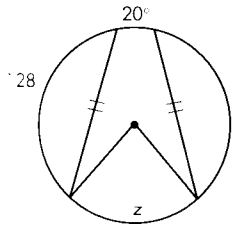
2. x = -?-



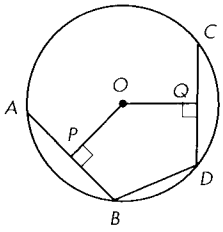
3.* y = -?-



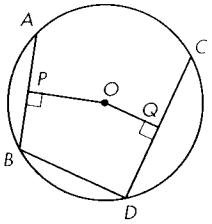
4. z = -?-



5. $AB = CD$
$PO = 8$ cm
OQ = -?-



6. $AB = 6$ cm $\quad OP = 4$ cm
$CD = 8$ cm $\quad OQ = 3$ cm
$BD = 6$ cm
What is the perimeter of $OPBDQ$?

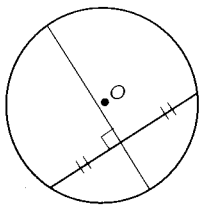


7. Use one of the circular objects you collected in Lesson 7.1 to trace a circle onto a clean sheet of paper. (Don't use your compass because then you'll know where the center is.) Use a compass and a straightedge to locate the center of the circle. Use the other circular object to trace a circle onto a patty paper and find the center by folding.

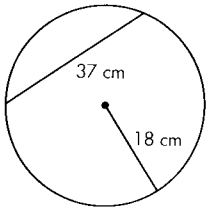
8. Use another circular object to trace a large minor arc. Locate by compass-and-straightedge construction a point on the arc equally distant from the arc's endpoints. Label it P.

9. Construct a triangle. Using the sides of the triangle as chords, construct a circle passing through all three vertices. Why does this seem familiar?

10.* What's wrong with this picture?



11. What's wrong with this picture?



12. Draw a circle and two chords of unequal length. Which is closer to the center of the circle, the larger chord or the smaller chord?

13. Draw two circles with different radii. In each circle, draw a chord of the same length. Draw the central angle determined by each chord. Which central angle is larger?