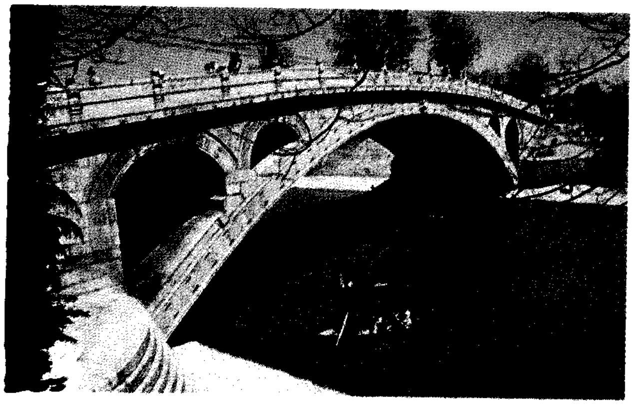
Lesson 7.4

Arcs and Angles

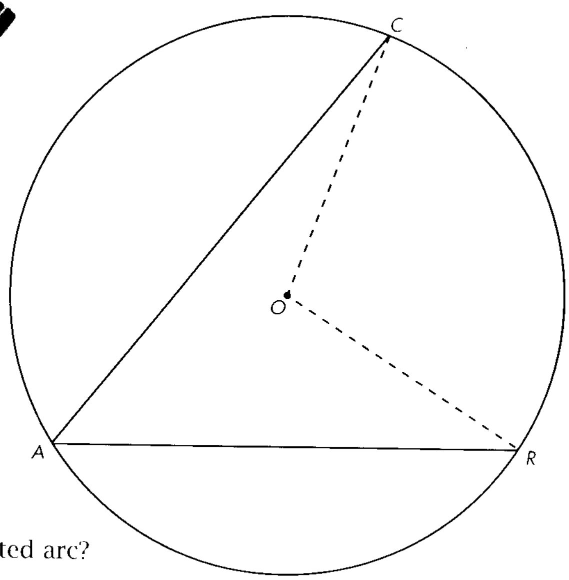


Many arches that you see in structures are semicircular, but the Chinese long ago discovered that arches don't have to be semicircular. The Zhaozhou bridge, shown at left, was completed in A.D. 605. It is the world's first stone arched bridge in the shape of a minor arc, predating similar structures by about 800 years.

In this lesson you'll discover properties of arcs and the angles associated with them.

Recall that the measure of a minor arc is the measure of its central angle. The measure of a major arc is 360° minus the measure of the minor arc making up the remainder of the circle. How does the measure of an inscribed angle compare with the measure of its intercepted arc? Let's investigate.

Investigation 7.4.1



Step 1 Measure $\angle COR$ with your protractor and determine $m\overset{\frown}{CR}$.

Step 2 Measure $\angle CAR$. How does $m\angle CAR$ compare with $m\overset{\frown}{CR}$?

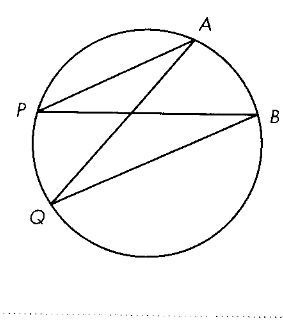
Step 3 Construct a circle of your own with an inscribed angle and its corresponding central angle.

Step 4 Measure the central angle. What is the measure of the intercepted arc?

Step 5 Measure the inscribed angle. How does the measure of the inscribed angle compare with the measure of its intercepted arc?

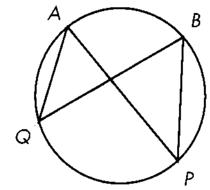
Compare your results with the results of others. State a conjecture.

C-68 The measure of an inscribed angle in a circle —?— (***Inscribed Angle Conjecture***).



In the figure at right, $\angle AQB$ and $\angle APB$ both intercept $\overset{\frown}{AB}$. Angles AQB and APB are both inscribed in $\overset{\frown}{APB}$. Angles AQB and APB appear congruent. Can you find angles inscribed in the same arc that are not congruent? Let's investigate.

Investigation 7.4.2



Step 1 Construct a large circle.

Step 2 Select two points on the circle. Label them A and B.

Step 3 Select a point P on the major arc and construct inscribed $\angle APB$.

Step 4 With your protractor, measure $\angle APB$.

Step 5 Select another point Q on major arc APB and construct inscribed $\angle AQB$.

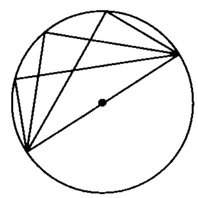
Step 6 Measure $\angle AQB$. How does the measure of $\angle AQB$ compare with the measure of $\angle APB$?

Repeat Steps 1-6 with points P and Q selected on the minor arc AB. Compare the measure of $\angle AQB$ with the measure of $\angle APB$. Compare your results with the results of others near you. Do you think you can find an angle inscribed in $\widehat{APB}$ that is not congruent to $\angle APB$? State your observations as a conjecture.

C-69 Inscribed angles that intercept the same arc are —?—.

Next you will discover a property of angles inscribed in semicircles.

Investigation 7.4.3



Step 1 Construct a large circle.

Step 2 Construct a diameter.

Step 3 Inscribe three angles in the same semicircle.

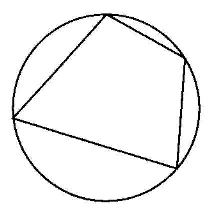
Step 4 Measure each angle with your protractor.

Compare your results with the results of others and make a conjecture.

C-70 Angles inscribed in a semicircle are —?—.

Now you will discover a property of the angles of a quadrilateral inscribed in a circle.

Investigation 7.4.4



Step 1 Construct a large circle.

Step 2 Construct an inscribed quadrilateral.

Step 3 Measure each of the four inscribed angles. Write the measure in each angle.

It is unlikely that any of the angles are congruent, but there is a special relationship between some pairs of angles. Compare your observations with the observations of those near you. State your findings as your next conjecture.

C-71 The —?— angles of a quadrilateral inscribed in a circle are —?—.

A quadrilateral inscribed in a circle is called a ***cyclic*** *quadrilateral.*

Next you will discover a property of arcs formed by parallel lines intersecting a circle.

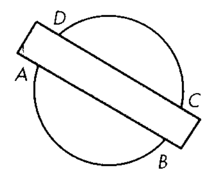
Investigation 7.4.5

Step 1 On a piece of patty paper, construct a large circle.

Step 2 Lay your straightedge across the circle so that its parallel edges pass through the circle. Draw lines along both edges of the straightedge. Label one arc AD and the other BC, as shown.

Step 3 Fold your patty paper to compare arcs AD and BC.

What can you say about arcs AD and BC? Repeat these steps, using either lined paper or another object with parallel edges. Compare your results with the results of others near you. State your observations as a conjecture.



C-72 Parallel lines intercept —?— arcs on a circle.

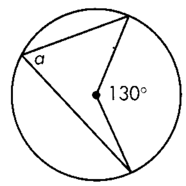
Take Another Look 7.4

1. Use a geometry computer program to construct a circle and an inscribed angle. Use a point on the circle besides the radius point for the vertex of the angle. Measure the angle and drag the vertex. What conjecture does this illustrate?
2. Extend Investigation 7.4.4 to see what other properties of cyclic quadrilaterals you can discover. A geometry computer program is a good tool for this activity, or use other tools of your choice.
3.* Choose one of the conjectures in this lesson and explain how it follows logically from earlier conjectures.
4.* State Conjecture 71 in "if-then" form. Then state the converse of the conjecture in "if-then" form. Is the converse also true?
5. On the surface of a sphere, investigate one or more of Conjectures 68 to 71.
6. Which of the following are always cyclic quadrilaterals: kites, isosceles trapezoids, rhombuses, rectangles, or squares? Explain why each is or is not cyclic.

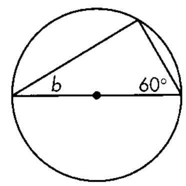
Exercise Set 7.4

Use your new conjectures to solve Exercises 1–16.

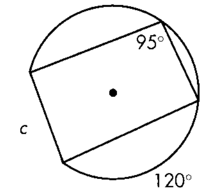
1. a = -?-



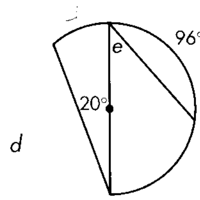
2. b = -?-



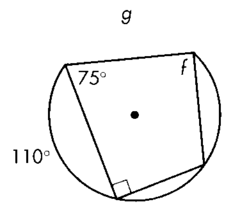
3.* c = -?-



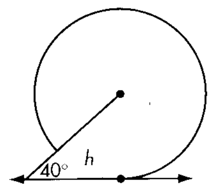
4. d = -?-
e = -?-



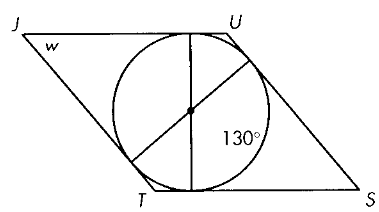
5. f = -?-
g = -?-



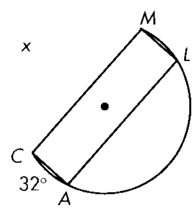
6.* h = -?-



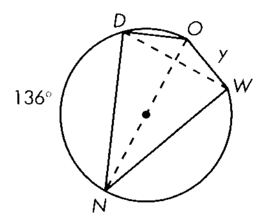
7. *JUST* is a rhombus.
w = -?-



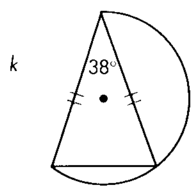
8. *CALM* is a rectangle.
x = -?-



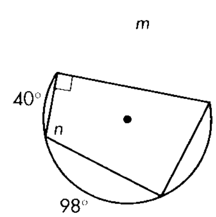
9. *DOWN* is a kite.
y = -?-



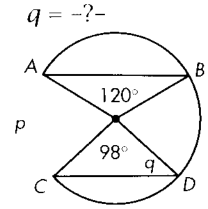
10. k = -?-



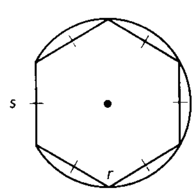
11. m = -?-
n = -?-



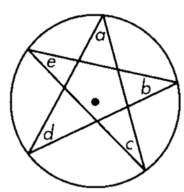
12.* $\overline{AB} \parallel \overline{CD}$
p = -?-
q = -?-



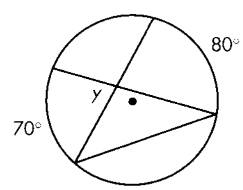
13. r = -?-
s = -?-



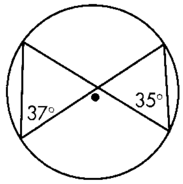
14.* What is the sum of
$a + b + c + d + e$?



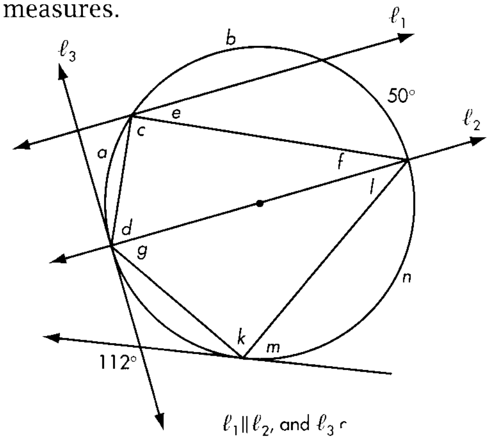
15.* y = -?-



16. What's wrong with this picture?



17. Find the lettered angle measures and arc measures.



$\ell_1 \parallel \ell_2$, and ℓ_3 ⌐

18. How can you find the center of a circle by using just the corner of a piece of paper?

19.* Chris Chisholm, while a high school student in Whitmore, California, used Conjecture 70 (Angles inscribed in a semicircle are . . . .) to discover a new and simpler way to find the orthocenter in a triangle. He noticed that if you construct a circle, using one of the sides of the triangle as the diameter of the circle, then you can immediately find an altitude to each of the other two sides! Try it. Use Chris's method to find the orthocenter of a triangle.

20.* The width of a view that can be captured in a photo depends **picture angle**. Most 35 mm cameras have a 46° picture an to take a photo of your class standing in one straight r Draw a line segment to represent the row. Draw a 46 angle on a piece of patty paper. Locate at least ei points where a 35 mm camera could be positi the students standing along the segment, fill picture as possible. What is the locus of all positions? What conjecture from this lesso activity illustrate?

21. Computer Activity Construct a circle and a di semicircles, and construct two chords to c semicircle. Locate the midpoint of each of two midpoints as the vertex of the right a visualizing this first. Sketch the locus by h the point on the circle and trace the locus

Improving Reasoning Ski

If the letter in the word *dinosaur* tha also found before the sixteenth lette horizontally. Otherwise print the wo letter after the first vowel.

-?-
-?-
-?-
-?-

r groups. Are the $\frac{C}{D}$ to 3 for every circle. We formula for finding the diameter is twice the umference in terms of the

the cir circle's If you circumfere accurate yo ratio will co π (pi), pronou favorite desser

circle, then there is a number e radius, then C = —?—

347

never ends and no pattern in k alphabet. Mathematicians xact value of the

d intrigued mathematicians d (4 3)² as their approximation 10. By A.D. 408, Chinese m alculated π to million