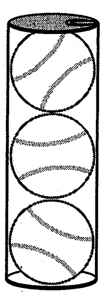
## Lesson 7.5

# The Circumference/Diameter Ratio

Here's a nice puzzle. One of two quarters remains motionless while the other rotates around it, never slipping and always tangent to it. When the rotating quarter has completed a turn around the stationary quarter, how many turns has it made around its own center point?

Did you guess one? Two? Three? The solution to the puzzle is not as obvious as it first appears. The best way of seeing the solution is to actually roll one coin around the other. Mark both coins with a felt tip pen or a pencil and try it.

The distance around a polygon is called the perimeter. The distance around a circle is called the **circumference.** In the puzzle above, one quarter is rolling along the circumference of the other quarter. In first thinking about the puzzle, you probably thought that because the circumferences of the two coins are the same, one coin would rotate once in its trip around the other. This was not the case, was it?

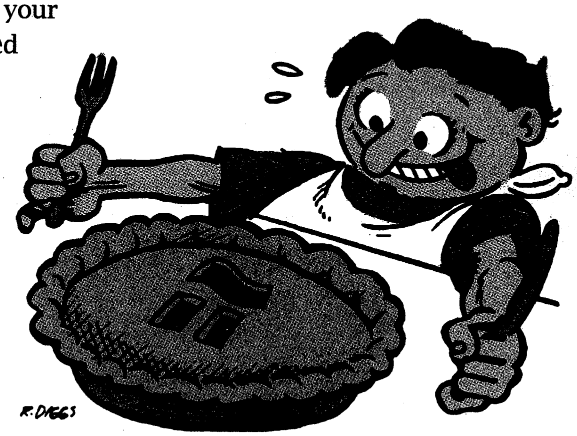


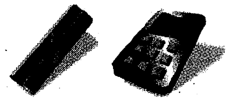
Here is another nice visual puzzle. Which is greater, the height of a tennis ball can or the circumference of the can? The height is approximately three tennis ball diameters tall. The diameter of the can is approximately one tennis ball in diameter. If you have a tennis ball can handy, try it. Wrap a string around the can to measure its circumference, then compare this measurement with the height of the can. Surprised?

In this second puzzle you should have discovered that the circumference of the can is greater than three diameters of the can. In this lesson you are going to discover (or perhaps rediscover) the relationship between the diameter and the circumference of every circle. Once you know this relationship, you can measure a circle's diameter and calculate its circumference.

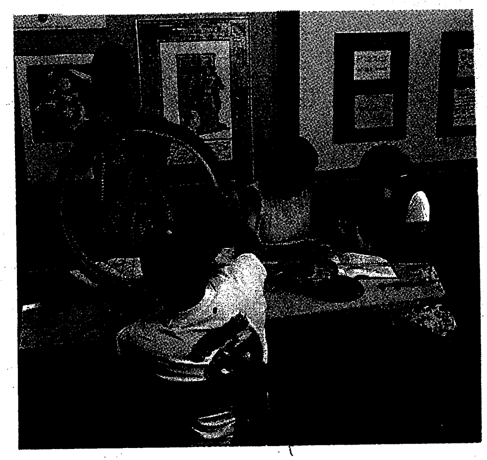
If you measure the circumference and diameter of a circle and divide the circumference by the diameter, you get a number slightly larger than three. The more accurate your measurements, the closer your ratio will come to a special number called $\pi$ (pi), pronounced like one of your favorite desserts.

In Investigation 7.5, you will experimentally determine an approximate value of $\pi$ by measuring circular objects and calculating their circumference/diameter ratio. Work in groups of four or five, sharing the tasks of measuring, recording, and calculating. Let's see how close you come to the actual value of $\pi$.



## Investigation 7.5 

For this investigation you will need the following special materials.



- Round objects you collected in Lesson 7.1 (the larger the objects the better)
- Meter stick or metric sewing tape
- Sewing thread or thin string to measure the circumference of each round object

**Step 1** With the thread and the meter stick (or the sewing tape), measure the circumference and diameter of each round object to the nearest millimeter (tenth of a centimeter).

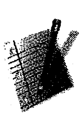
**Step 2** Make a table similar to the one below and record the circumference ($C$) and diameter ($D$) measurements for each round object.

| | | | | |
|---|---|---|---|---|
| Name of object | -?- | -?- | -?- | -?- |
| Circumference ($C$) | -?- | -?- | -?- | -?- |
| Diameter ($D$) | -?- | -?- | -?- | -?- |
| $C/D$ | -?- | -?- | -?- | -?- |

**Step 3** Calculate $\frac{C}{D}$ and record the answers in your table.

**Step 4** Calculate the average of your $\frac{C}{D}$ results.

Compare the average of your $\frac{C}{D}$ results with the $\frac{C}{D}$ averages of other groups. Are the $\frac{C}{D}$ answers close? You should now be convinced that $\frac{C}{D}$ is very close to 3 for every circle. We define the ratio: $\frac{C}{D} = \pi$. If you solve this formula for $C$, you get a formula for finding the circumference of a circle in terms of the diameter. Because the diameter is twice the radius ($D = 2r$), you also can get a formula for finding the circumference in terms of the radius. State your conjecture.



**C-73** If $C$ is the circumference and $D$ is the diameter of a circle, then there is a number $\pi$ such that $C =$ —?—. Because $D = 2r$ where $r$ is the radius, then $C =$ —?— (***Circumference Conjecture***).

The number $\pi$ is an irrational number. Its decimal form never ends and no pattern in it ever repeats. The symbol $\pi$ is a letter from the ancient Greek alphabet. Mathematicians began using it in the eighteenth century to represent the exact value of the circumference/diameter ratio.

Perhaps no other number has more fascinated and intrigued mathematicians throughout history. Mathematicians in ancient Egypt used $(4/3)^4$ as their approximation of $\pi$. Early Chinese and Hindu mathematicians used $\sqrt{10}$. By A.D. 408, Chinese mathematicians were using 355/113. Today, computers have calculated $\pi$ to millions of decimal places.

Such accurate approximations for $\pi$ have been more of intellectual interest than for practical purposes. Still, what do you think a carpenter would say if you asked her to cut a board so that it was $3\pi$ feet long? Most calculators have a $\pi$ button that gives $\pi$ to eight or ten decimal places. You can use it for most calculations, then round your answer to a specified decimal place. If your calculator doesn't have a $\pi$ button, or if you ever find yourself without access to a calculator, use the value 3.14 for $\pi$. If you're asked for an exact answer instead of an approximation, state your answer in terms of $\pi$.

How do you use the Circumference Conjecture? Let's look at two examples.

**Example A**

If a circle has a circumference of $12\pi$ meters, what is the radius?

$$C = 2\pi r$$
$$12\pi = 2\pi r$$
$$r = 6$$

The radius is 6 meters.

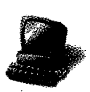
**Example B**

If a circle has a diameter of 3.0 meters, what is the circumference? Use a calculator and state your answer to the nearest 0.1 meter.

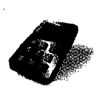
$$C = \pi D$$
$$C = \pi(3.0)$$

The circumference is about 9.4 meters.

## Take Another Look 7.5



1. Use a geometry computer program to construct a circle. Construct a diameter. Measure the circumference and the diameter. Calculate the ratio of the circumference to the diameter. Change the size of your circle (thus changing the size of both the diameter and the circumference) and observe the ratio of the circumference to the diameter. Does it change? Explain how this confirms the Circumference Conjecture.
2. Locate high-quality spherical balloons, bow calipers, string, and meter sticks. Blow one large breath into a balloon. With the string, measure the balloon's circumference. With the bow calipers, measure the balloon's diameter. Repeat these two steps after blowing into the balloon a second full breath, a third full breath, until the balloon is near breaking. Find the ratio—the circumference to the diameter—for each pair of measurements. Does this confirm the Circumference Conjecture? Explain.



3. Use graph paper or a graphing calculator to graph the data collected from Investigation 7.5 or from Activities 1 and 2 in this Take Another Look. Graph the diameter on the $x$-axis and the circumference on the $y$-axis. What is the slope of the best-fit line through the data points? Does this confirm the Circumference Conjecture? Explain.

## Exercise Set 7.5

Use the Circumference Conjecture to solve Exercises 1–13. In Exercises 1–8, do not use an approximation for $\pi$.

1. If $r = 5$ cm, find $C$.
2. If $C = 5\pi$ cm, find $D$.
3. If $C = 24$ m, find $r$.
4.* If $D = 5\pi$ m, find $C$.

5. If a circle has a diameter of 12 cm, what is its circumference?

6. If a circle has a circumference of $46\pi$ m, what is its diameter?

7.* If a circle is inscribed in a square with a perimeter of 24 cm, what is the circumference of the circle?

8. If a circle with a circumference of $16\pi$ in. is circumscribed about a square, what is the length of a diagonal of the square?

In Exercises 9–13, use a calculator. Round your answer to the nearest 0.1 unit. Use the symbol ≈ to show that your answer is an approximation.

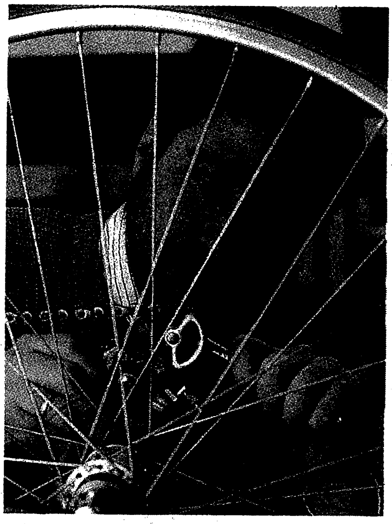
9. If $D = 5$ cm, find $C$.

10. If $r = 4$ cm, find $C$.

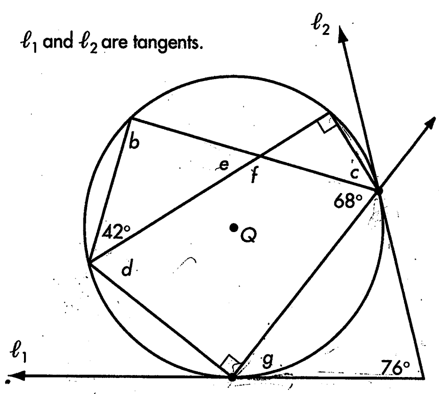
11. If $r = 7$ m, find $C$.

12.* If $C = 44$ m, find $r$.

13. What's the circumference of a bicycle wheel with a 27-inch diameter?

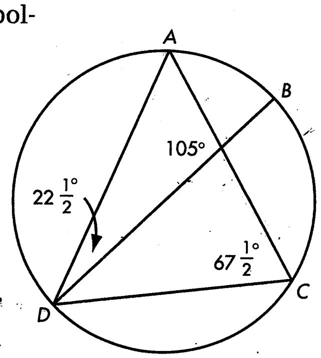


14. Trace the figure below. Calculate each lettered angle measure.




15. According to the *Atlas of the Environment* (1992), the rivers and lakes of most of the developing world are clogged with a mixture of agricultural chemicals, industrial toxins, and untreated urban sewage. In 1970, the Rhine river was so polluted that a German citizen was able to partially develop a roll of film in it. The biggest polluter of our rivers is agricultural land packed with pesticides and fertilizers.

Of the primary sources of pollution, agriculture contributes 64%. Resources extraction, such as mining, contributes 9% of the pollution, forestry contributes another 6%, urban runoff another 5%, hydromodification 4%, construction 2%, land-disposal 1%, and the remaining 9% is contributed by a variety of natural and human pollution. Create a circle graph that shows the distribution of the primary sources of river pollution.




16. What is the probability that a flea strolling along the circle shown at right will randomly stop on either $\widehat{AB}$ or $\widehat{CD}$? (Because you're probably not an expert in flea behavior, assume the flea will stop exactly once.)