

Lesson 9.2

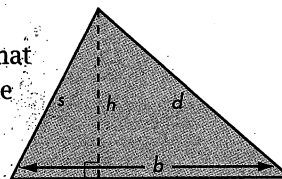
Areas of Triangles, Trapezoids, and Kites

In this lesson you will physically demonstrate the formula for the area of triangles and the formula for the area of trapezoids.

Investigation 9.2.1



On a piece of heavy paper or cardboard, draw a triangle that measures about 6 cm in height by 10 cm at the base. Label the triangle as shown at right. Cut out the triangle and make a copy of it. Cut out the copy. Arrange the two triangles to form a figure for which you already have an area formula.



Is the area of this figure equal to the sum of the areas of the two triangles?

Remember, if two polygons are congruent, their areas are equal. What is the area of one of the triangles? Make a conjecture. Write a brief description in your notebook of how you arrived at the formula. Include an illustration.

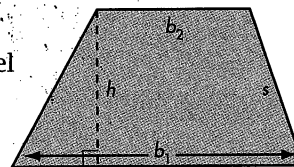


C-83 The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where A is the area, b is the length of the base, and h is the height of the triangle (*Triangle Area Conjecture*).

Investigation 9.2.2



As you remember from Chapter 6 (yeah, sure!), the parallel sides of the trapezoid are called the bases. The **altitude** of a trapezoid is a segment (or the length of a segment) from one base to the other, perpendicular to both. The **height** of a trapezoid is the length of an altitude.



On a piece of heavy paper or cardboard, construct a trapezoid that measures about 6 cm in height by 10 cm. Construct an altitude from one of the endpoints of the smaller base to the larger base. Label the trapezoid as shown above right. Cut out the trapezoid and trace it onto another piece of paper. Cut out the copy. Arrange the two trapezoids to form a figure for which you already have an area formula.

What type of polygon is created by the two trapezoids placed next to each other? What is the height of this figure? What is the length of the base? What is the area? What is the area of one of the trapezoids? State a conjecture.



C-84 The area of a trapezoid is given by the formula $A = \frac{1}{2}(b_1 + b_2)h$, where A is the area, b_1 and b_2 are the lengths of the two bases, and h is the height of the trapezoid (*Trapezoid Area Conjecture*).

Investigation 9.2.3



Now you are going to create and carry out your own investigation to discover a formula for the area of a kite. Do you recall some of the properties of a kite? These kite properties may be helpful: The diagonal connecting the vertex angles of a kite divides the kite into two congruent triangles; the diagonal connecting the nonvertex angles of a kite divides the kite into two isosceles triangles; and the diagonals of a kite are perpendicular to each other and thus divide the kite into four right triangles. You're on your own. Investigate.

Compare your results with the results of others near you. State a conjecture.



C-85 The area of a kite is given by the formula $A = \frac{1}{2}d_1d_2$ (Kite Area Conjecture).

Take Another Look 9.2

Try one or more of these follow-up activities.



- Use a geometry computer program to construct a triangle or a trapezoid whose perimeter you can change without changing the area.
- Provide a logical argument demonstrating that when you put two copies of the same trapezoid together, as you did in Investigation 9.2.2, the figure is a parallelogram.
- True or false? The area of a triangle is equal to half the perimeter times the radius of the inscribed circle. Support your conclusion with a convincing argument.
- * Find four more methods for arriving at the formula for the area of a trapezoid. Consider ways of dividing the trapezoid into pieces (triangles, rectangles, parallelograms) and then rearranging the pieces into shapes for which you already know the area. See Exercises 27 and 28 for two methods.

Exercise Set 9.2

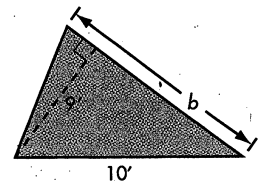
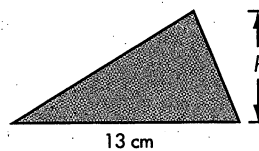
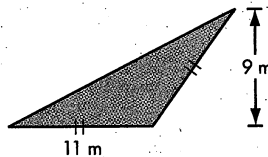
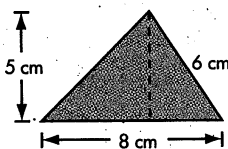
Use your new area conjectures to solve Exercises 1-16. In Exercises 9 and 10, the quadrilaterals are kites. In Exercises 11-16, the quadrilaterals are trapezoids.

1. $A = \frac{1}{2}bh$

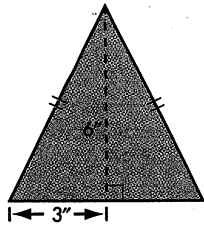
2.* $A = \frac{1}{2}(b_1 + b_2)h$

3. $A = \frac{1}{2}bh$
 $h = \frac{2A}{b}$

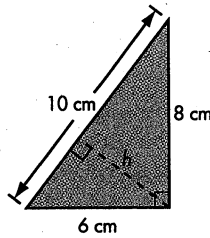
4.* $A = \frac{1}{2}bh$
 $b = \frac{2A}{h}$



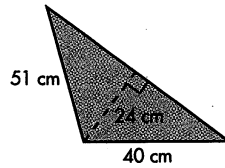
5. $A = ?-$



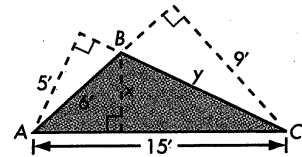
6. $h = ?-$



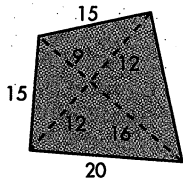
7. If the area is 924 cm^2 , what is the perimeter?



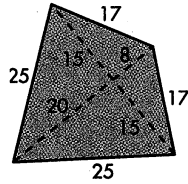
8.* $x = ?-$
 $y = ?-$



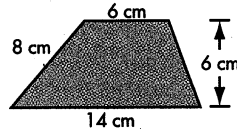
9. $A = ?-$



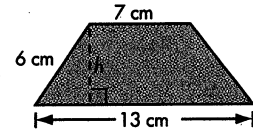
10. $A = ?-$



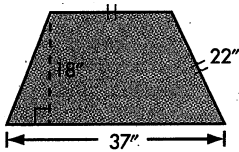
11. $A = ?-$



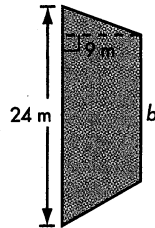
12. $A = 50 \text{ cm}^2$
 $h = ?-$



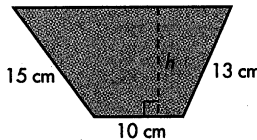
13. $A = ?-$



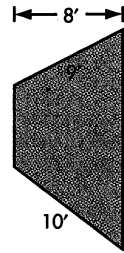
14. $A = 180 \text{ m}^2$
 $b = ?-$



15. If $A = 204 \text{ cm}^2$ and $P = 62 \text{ cm}$, then $h = ?-$.



16.* $A = 84 \text{ sq ft}$
 $P = ?-$



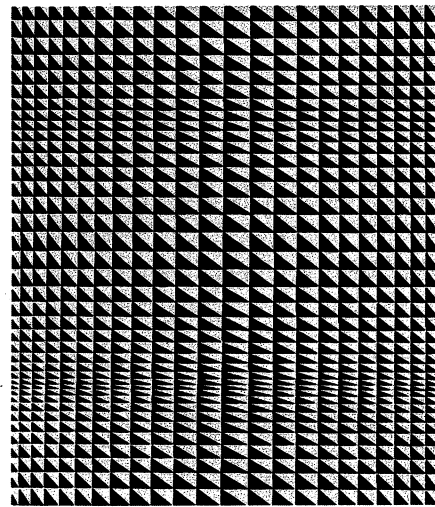
17.* Sketch and label two different triangles, each with an area of 54 cm^2 .

18.* Sketch and label two different trapezoids, each with an area of 56 cm^2 .

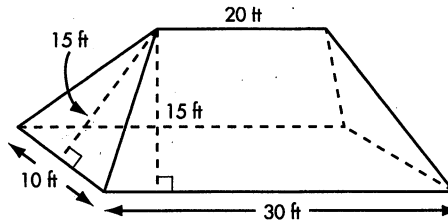
19. Sketch and label two different kites, each with an area of 1092 cm^2 .

20. Sketch and label a triangle and a trapezoid with equal areas and equal heights. How does the base of the triangle compare with the two bases of the trapezoid?

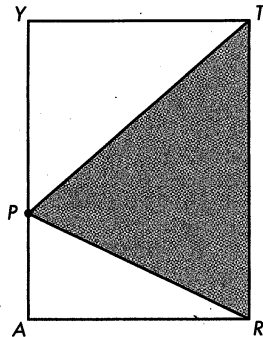
21. Bridget Riley's painting *Straight Curve* consists of columns and rows of triangles with varying heights and bases. What's the optical effect of this variation? What do you think the title of the painting means? What fraction of the total area do the dark triangles occupy?



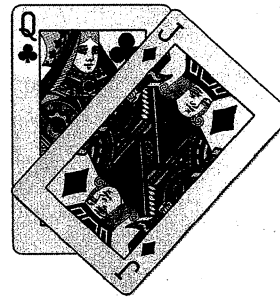
22. The roof on Crystal's cottage is formed by two congruent trapezoids and two congruent isosceles triangles. She wants to put new shingles on her roof. Each shingle will cover 0.75 ft^2 of area. How many shingles should she buy?



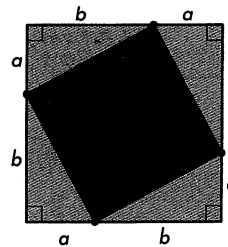
23. P is an arbitrary point on side AY of rectangle $ARTY$. The shaded area is what fraction of the area of the rectangle?



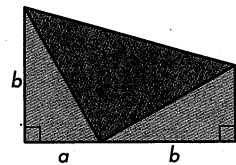
24. One playing card is placed over another, as shown. Is the top card covering half, less than half, or more than half of the bottom card? Explain.



25. The square region on the right has been divided into four congruent right triangles and a quadrilateral in the center. Is the quadrilateral a square? How do you know? What is the sum of the areas of the four triangles? What is the area of the quadrilateral in the center?



26. A trapezoid has been created with two congruent right triangles and an isosceles triangle, as shown on the right. Is the isosceles triangle a right triangle? How do you know? Find the area of the trapezoid two ways: by using the trapezoid area formula and then by finding the sum of the areas of the three triangles.



- 27.* Divide a trapezoid into two triangles. Use algebra to derive the formula for the area of the trapezoid by expressing the area of each triangle algebraically and finding their algebraic sum.
- 28.* Divide a trapezoid into a parallelogram and a triangle. Use algebra to derive the formula for the area of the trapezoid by expressing the area of the triangle and the area of the parallelogram algebraically and finding their algebraic sum.