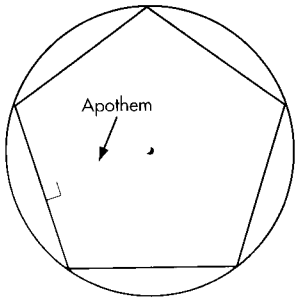
Lesson 9.4

Areas of Regular Polygons

In this lesson you will discover a formula for finding the area of any regular polygon. A regular pentagon, a regular hexagon, and a regular octagon are shown in Investigation 9.4, inscribed in circles. You can divide a regular polygon of n sides into n congruent isosceles triangles by drawing the radii from the center of the circumscribed circle to each vertex of the polygon. Then you can find the area of one isosceles triangle and multiply by the number of isosceles triangles in the polygon (n) to find the area of the regular polygon of n sides.

*An **apothem** of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon.*

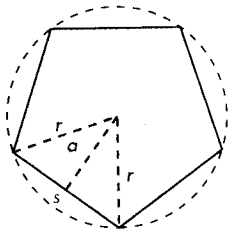


You may also refer to the length of the segment as the apothem.

In Investigation 9.4 you will calculate the area of regular polygons. Then you will write a formula for the area of a regular polygon.

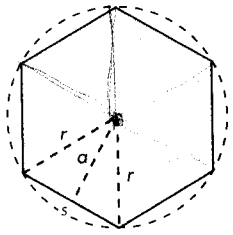
Investigation 9.4

Let the length of the apothem of each regular polygon equal a and let the length of each side of the regular polygon equal s.



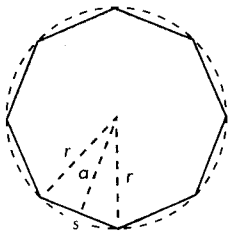
Area of a regular pentagon

$A = (\frac{1}{2})as(5)$



Area of a regular hexagon

$A = (\frac{1}{2})as(6)$



Area of a regular octagon

$A = (\frac{1}{2})as(8)$

Copy the table below. Then find the area of each regular polygon in the table in terms of a and s.

Number of sides	3	4	7	9	10	11	12	...	n
Area of polygon	-?-	-?-	-?-	-?-	-?-	-?-	-?-	...	-?-

Your last entry in the table can be restated as your next conjecture.

C-86 The area of a regular polygon is given by the formula —?—, where A is the ar is the apothem, s is the length of each side, and n is the number of sides of t regular polygon. Because the length of each side times the number of sides is perimeter, $sn = p$. The formula can also be written as $A = (\frac{1}{2})a$ -?- (***Regular Polygon Area Conjecture***).

Take Another Look 9.4

Try one or more of these follow-up activities.

1. Use a geometry computer program to demonstrate the Regular Polygon Area Conjecture.
2.* Create another method for arriving at the formula for the area of a regular polygon. Consider ways of dividing a regular polygon into isosceles triangular pieces and rearranging the pieces into shapes for which you already know the area.
3. True or false? The area of any cyclic quadrilateral is equal to half the perimeter times the radius of the inscribed circle. Support your conclusion with a convincing argument.
4.* Can you use the Regular Polygon Area Conjecture to arrive at a formula for the area of a circle? Explain your reasoning. Use a series of diagrams to help.

Exercise Set 9.4

In Exercises 1–4, use the Regular Polygon Area Conjecture to find the area of each regular polygon, accurate to the nearest square centimeter. The apothem is a, s is the length of a side, and p is the perimeter.

1. Pentagon: $a \approx 3$ cm and $s \approx 4.4$ cm
2. Decagon: $a \approx 5.7$ cm and $s \approx 3.7$ cm
3. Octagon: $a \approx 12.1$ cm and $p \approx 80$ cm
4. Nonagon: $p \approx 63$ cm and $a \approx 9.6$ cm
5.* Use your compass and straightedge to construct a regular hexagon with sides that measure 4 cm. Use the Regular Polygon Area Conjecture and a centimeter ruler to approximate the hexagon's area.
6.* Use your geometric tools to draw a regular pentagon with an apothem that measures 4 cm. Use the Regular Polygon Area Conjecture and a centimeter ruler to approximate the pentagon's area.
7. Use your geometric tools to draw a regular octagon with sides that measure 4 cm. Use the Regular Polygon Area Conjecture and a centimeter ruler to approximate the octagon's area.

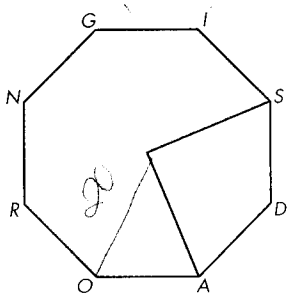
Exercises 8, 11, and 12, express each answer accurate to the nearest square centimeter. The
othem is a, p is the perimeter, and n is the number of sides of the regular polygon.

Find the area of a regular polygon with $a \approx 12$ cm and $p \approx 81.6$ cm.

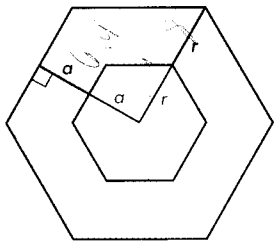
Find the perimeter of a regular polygon to the nearest tenth of a meter if $a \approx 9$ m and $A \approx 259.2$ m^2.

. Find the length of each side of a regular polygon to the nearest foot if $a \approx 80$ ft, $n = 20$, and $A \approx 20{,}000$ sq ft.

. Find the shaded area of the regular octagon *ROADSIGN*. The apothem measures about 20 cm. Segment *GI* measures about 16.6 cm.



12.* Find the shaded area of the regular hexagonal donut. The shorter apothem and shorter sides are half as long as the longer apothem and longer sides.
$a \approx 6.9$ cm
$r \approx 8$ cm



Exercises 13–15, graph each line, then find the area bounded by the x-axis, the y-axis, d each line.

.* $y = \frac{1}{2}x + 5$
$y = -2x + 10$

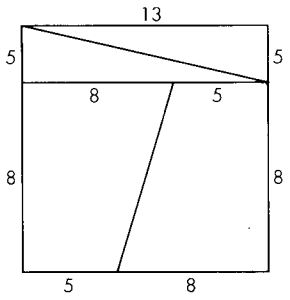
14. $y = \frac{-1}{3}x + 6$
$y = \frac{-1}{3}x + 12$

15. $y = \frac{-3}{2}x - 3$
$y = \frac{1}{4}x + 4$

. Use a rectangle diagram to show the factors of $x^2 + 11x + 28$.

. Sketch and label a rectangle that demonstrates the following algebraic expression.
$(x + y + 1)(x + y + 3) = x^2 + 2xy + y^2 + 4x + 4y + 3$

.* Kelvin divided a square with an area of 169 square units (13×13) into two trapezoids and two right triangles, as shown. But when he rearranged the four parts into a rectangle, he got an area of 168 square units (8×21)! So he then rearranged the four parts into a triangle (base measuring 16 and height measuring 21), again with an area of 168 square units! What is happening? Try it.



On graph paper, draw a square like that shown, cut out the pieces, and rearrange them into a 8×21 rectangle and then into a triangle. Help Kelvin. He wants to know where the missing area is going.

. **Computer Activity** Construct a triangle. Construct the three medians. Compare the areas of the six small triangles formed. Make a conjecture. Support your conjecture with a convincing argument.