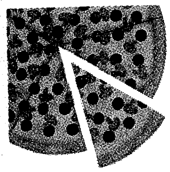


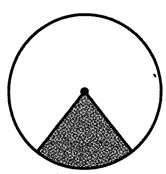
Lesson 9.6

Any Way You Slice It

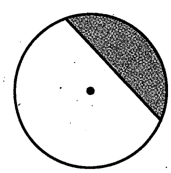
Cut my pie into four pieces—I don't think I could eat eight.
— Yogi Berra

Its makers claimed this was the world's largest slice of pizza.

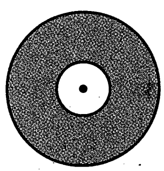
In Lesson 9.5, you discovered a formula for calculating the area of a circle. In this lesson you will discover how to calculate the area of different portions of a circle. If you were told to cut a slice of pizza, your slice would probably be in the shape of a sector of a circle. If for some reason you were permitted only one straight cut with your knife, your slice would probably be in the shape of a segment of a circle. If your sister or brother didn't like the crusts, she or he might cut out a circular slice from the center of the pizza. The shape that remains is called an annulus.



Sector of a circle



Segment of a circle



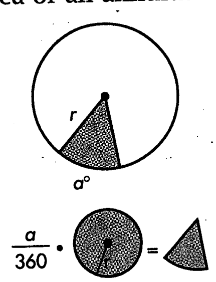
Annulus

A ***sector of a circle*** *is the region between two radii of a circle and the included arc.*

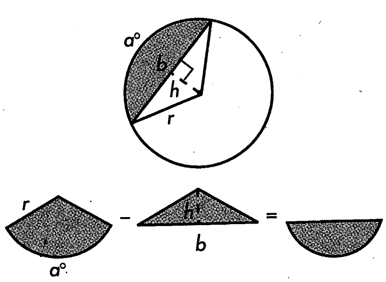
A ***segment of a circle*** *is the region between a chord of a circle and the included arc.*

An ***annulus*** *is the region between two concentric circles.*

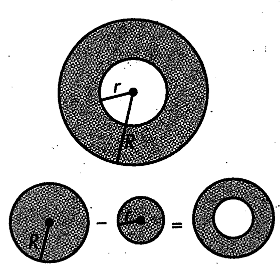
In this lesson you will solve problems that include finding the areas of sectors of circles, segments of circles, and annuluses. "Picture equations" are helpful when you are trying to visualize the area of some of these regions. The picture equations below show you how to find the area of a sector of a circle, the area of a segment of a circle, and the area of an annulus.



Area of a sector
$\left(\frac{a}{360}\right) \times \pi r^2$

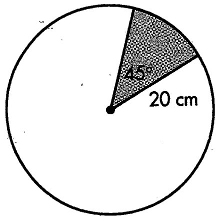


Area of a segment
$\left(\frac{a}{360}\right) \times \pi r^2 - \frac{1}{2}bh$



Area of an annulus
$\pi R^2 - \pi r^2$

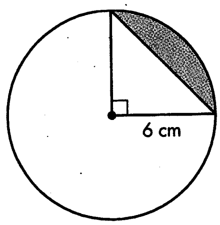
Example A



Find the area of the sector.

Because $r = 20$ cm, the area of the whole circle is $\pi 20^2$, or 400π cm^2. Because $\frac{45}{360} = \frac{1}{8}$, the area of the sector is $(\frac{1}{8})(400\pi \text{ cm}^2)$, or 50π cm^2.

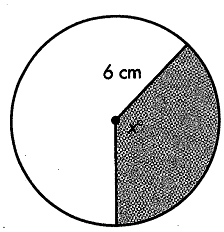
Example B



Find the area of the segment.

Because $r = 6$ cm, the area of the circle is $\pi 6^2$, or 36π cm^2. Because the sector is $\frac{1}{4}$ of the circle, the area of the sector is $(\frac{1}{4})(36\pi \text{ cm}^2)$, or 9π cm^2. Because the triangle is a right triangle, its area is $(\frac{1}{2})(6)(6)$, or 18 cm^2. Therefore the area of the segment is the area of the sector (9π cm^2) less the area of the triangle (18 cm^2), or $(9\pi - 18)$ cm^2.

Example C

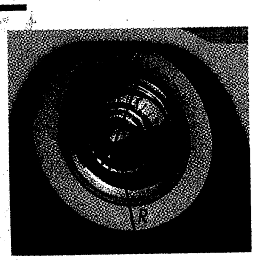


Find x. The shaded area is 14π cm^2, and the radius is 6 cm. The sector's area is $(\frac{x}{360})$ of the circle's area.

Therefore
$$14\pi = \left(\frac{x}{360}\right)(36\pi)$$
$$\frac{(360)(14\pi)}{(36\pi)} = x$$
$$x = 140.$$

The central angle measures 140°.

Example D



Find the area of the whitewall portion of the tire (assuming it doesn't bulge much and lies in a plane). $R = 13$ in. and $r = 9$ in. The area of the annulus is equal to the area of the larger circle (169π sq in.) less the area of the smaller circle (81π sq in.). Therefore the area of the annulus is $169\pi - 81\pi$, or 88π sq in.

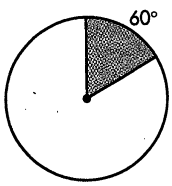
Take Another Look 9.6

A friend tells you she's found another way to find the area of an annulus: First find the average of the circumferences of the two circles that form the annulus. Then multiply that average by the thickness of the annulus (the difference between the radii) to find the area. Is your friend right? Draw a picture and use algebra to show why this does or doesn't work.

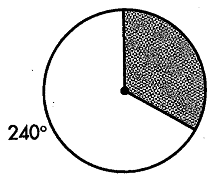
Exercise Set 9.6

In Exercises 1–8, find the shaded area. The radius of each circle is r. If two circles are shown, r is the radius of the smaller and R is the radius of the larger. All given measurements are in centimeters.

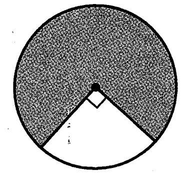
1. $r = 6$



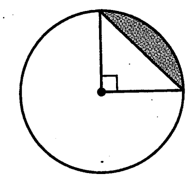
2. $r = 8$



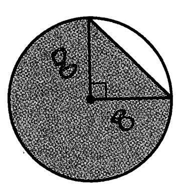
3.* $r = 16$



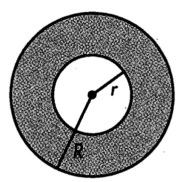
4. $r = 2$



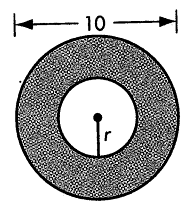
5. $r = 8$



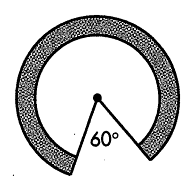
6.* $R = 7$
$r = 4$



7. $r = 2$

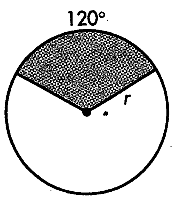


8. $R = 12$
$r = 9$

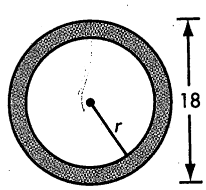


In Exercises 9 and 10, find the radius. In Exercises 11 and 12 find $m\angle ABC$.

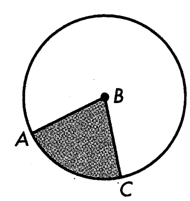
9.* The shaded area is 12π cm^2.



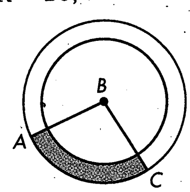
10. The area of the annulus is 32π cm^2.



11. The shaded area is 120π cm^2.
$r = 24$ cm



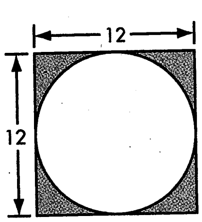
12. The shaded area is 10π cm^2.
$R = 10$, $r = 8$



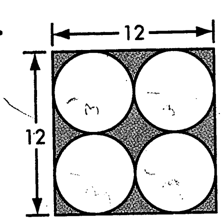
13. Suppose the pizza slice in the photo at the beginning of this lesson is a sector with a 36° angle in a circle with a radius of 20 ft. If a can of tomato sauce will cover 3 ft^2 of pizza, how many cans would be required to cover this slice?

In Exercises 14–17, what is the shaded area in each figure? In Exercises 15–17, the circles are externally tangent. The area of the circle or circles in each figure is what percentage of the area of the square? All given measurements are in centimeters.

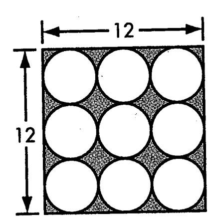
14.



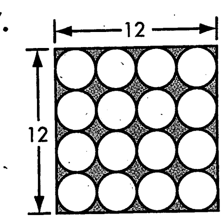
15.



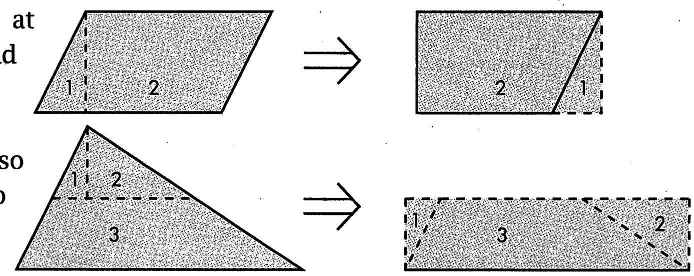
16.



17.



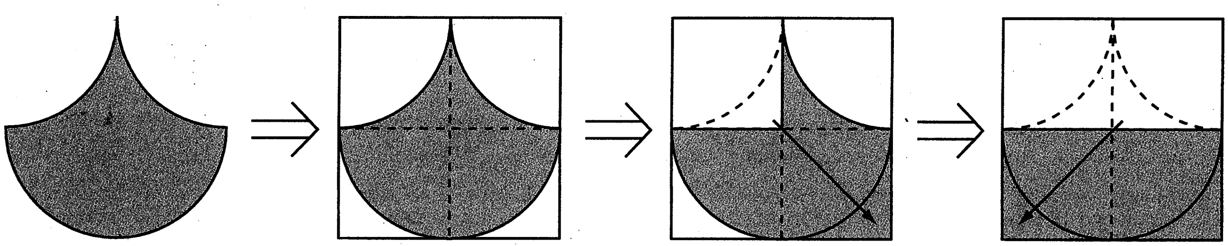
18. In Investigation 9.1.2, you cut a right triangle from one end of a parallelogram and reassembled the two parts into a rectangle. You can also cut a triangular region into three parts and reassemble them into a rectangular region, as shown at right. In doing this cutting and reassembling, you **rectify** (make into a rectangle) the parallelogram and the triangle. You can also cut any trapezoidal region into three parts and reassemble them into a rectangular region. Try it.



19. The fascination with dissecting geometric shapes and reassembling them into rectangles can be traced back to early Greek mathematicians. A famous "impossible problem" from ancient Greece is the squaring of the circle problem: Construct with a compass and a straightedge a square having an area equal to that of a given circle.

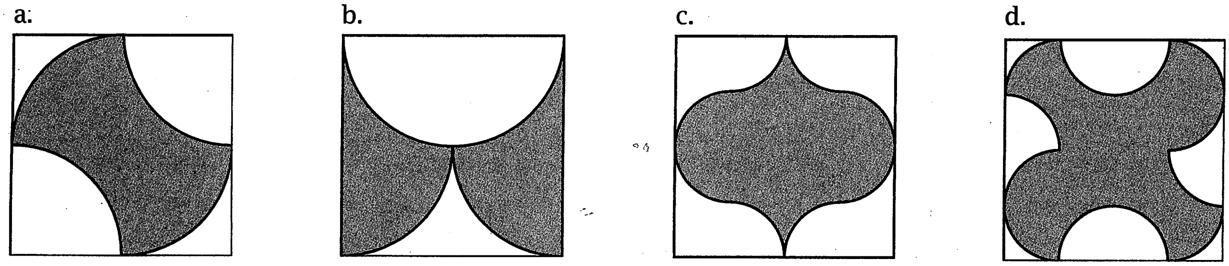
It wasn't until the nineteenth century that this problem was proved impossible to solve with only a compass and a straightedge. In the time it took to prove this, many geometric discoveries were made while attempting to "square the circle." Even today, professional mathematicians still get mail from geometric explorers who think they have squared the circle.

A fifteenth-century geometric investigator who was fascinated with this problem was Leonardo da Vinci (1452–1519). In attempting to solve the quadrature problem, Leonardo and others were successful in rectifying some special shapes made up of parts of circles. The illustrations below demonstrate how to rectify the pendulum.



In a series of diagrams, demonstrate how to rectify each figure.

a. b. c. d.



20. Can you reverse the process you carried out in Exercise 19? On a piece of graph paper, draw a rectangle that measures 12 units by 6 units. Use your compass to divide it into at least four parts, then rearrange the parts into a curved figure. Draw the outline of the curved figure onto another piece of graph paper.