

6 EXAMPLE Solving a Higher-Degree Polynomial Equation

Solve $x^4 - x^2 = 12$.

$$x^4 - x^2 = 12$$

$$x^4 - x^2 - 12 = 0$$

Rewrite so one side of the equation is equal to zero.

$$(x^2)^2 - (x^2) - 12 = 0$$

Write in the form of a quadratic expression. Think of the expression as $a^2 - a - 12$, which factors as $(a - 4)(a + 3)$.

$$(x^2 - 4)(x^2 + 3) = 0$$

$$(x - 2)(x + 2)(x^2 + 3) = 0$$

$$x = 2 \quad \text{or} \quad x = -2 \quad \text{or} \quad x^2 = -3$$

Use the Factor Theorem.

$$x = \pm 2 \quad \text{or} \quad x = \pm\sqrt{-3}$$

Solve for x .

$$x = \pm 2 \quad \text{or} \quad x = \pm i\sqrt{3}$$

Simplify.

• The solutions are $2, -2, i\sqrt{3}$, and $-i\sqrt{3}$.

✓ **Check Understanding** 6 Solve $x^4 + 11x^2 + 18 = 0$.

EXERCISES

For more practice, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Examples 1 and 2
(pages 321 and 322)



Solve each equation by graphing. Check your answers.

1. $x^3 - 4x^2 - 7x = -10$

2. $3x^3 - 6x^2 - 9x = 0$

3. $4x^3 - 8x^2 + 4x = 0$

4. $6x^2 = 48x$

5. $x^3 + 3x^2 + 2x = 0$

6. $2x^3 + 5x^2 = 7x$

7. $4x^3 = 4x^2 + 3x$

8. $2x^4 - 5x^3 - 3x^2 = 0$

9. $x^2 - 8x + 7 = 0$



10. **Savings** The polynomial $1600x^3 + 1200x^2 + 800x$ represents your savings, with interest, from a summer job after three years. The annual interest rate equals $x - 1$. Find the interest rate needed so that you will have \$4000 at the end of three years.



11. **Geometry** The volume V of a container is modeled by the function $V(x) = x^3 - 3x^2 - 4x$. Let $x, x + 1$, and $x - 4$ represent the width, the length, and the height respectively. The container has a volume of 70 ft^3 . Find the container's dimensions.

Example 3
(page 322)

Factor each expression.

12. $x^3 + 64$

13. $x^3 - 1000$

14. $125x^3 - 27$

Example 4
(page 323)

Solve each equation.

15. $x^3 - 27 = 0$

16. $x^3 + 64 = 0$

17. $x^3 - 125 = 0$

18. $2x^3 + 2 = 0$

19. $8x^3 - 1 = 0$

20. $64x^3 + 8 = 0$

Example 5
(page 323)

Factor each expression.

21. $x^4 - 8x^2 + 7$

22. $x^4 + 8x^2 - 20$

23. $x^4 - 7x^2 + 12$

24. $x^4 - 5x^2 + 4$

25. $x^4 - 1$

26. $4x^4 - 6x^2 + 2$

Example 6
(page 324)

Solve each equation.


27. $x^4 - 10x^2 + 9 = 0$ 28. $x^4 - 8x^2 + 16 = 0$ 29. $x^4 - 12x^2 - 64 = 0$
 30. $x^4 + 7x^2 - 18 = 0$ 31. $x^4 + 4x^2 - 12 = 0$ 32. $x^4 + 8x^2 + 15 = 0$

B Apply Your Skills

 Solve each equation by graphing. Where necessary, round to the nearest hundredth.

33. $x^3 - x^2 - 6x - 4 = 0$ 34. $2x^4 + 18x^3 = 0$
 35. $x^4 + 2x^3 - 7x^2 - 8x = -12$ 36. $x^4 + x^3 = 4x^2 + 4x - 5$
 37. $x^3 + 13x = 10x^2$ 38. $x^3 - 6x^2 + 6x = 0$ 39. $12x^3 = 60x^2 + 75x$

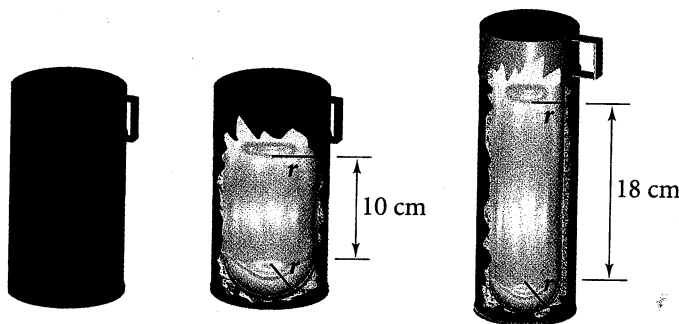
40. The product of three consecutive integers $n - 1$, n , and $n + 1$ is 210. Write and solve an equation to find the numbers.

 41. **Design** The chamber in each container below consists of a cylinder on top of a hemisphere. Each chamber holds 500 cm^3 . Find the radius of each chamber.



Real-World Connection


The vacuum bottle, or Dewar flask, was invented by the chemist James Dewar for storing liquefied gases.




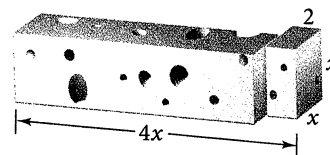
Solve each equation.


42. $125x^3 + 216 = 0$ 43. $81x^3 - 192 = 0$ 44. $x^4 - 64 = 0$
 45. $-2x^4 + 46x^2 = -100$ 46. $27 = -x^4 - 12x^2$ 47. $x^5 - 5x^3 + 4x = 0$
 48. $x^4 - 100 = 0$ 49. $5x^3 = 5x^2 + 12x$ 50. $64 - x^3 = 0$
 51. $x^3 - 6x^2 + 6x = 0$ 52. $2x^3 = 5x^2 + 12x$ 53. $3x^4 + 12x^2 - 15 = 0$
 54. $x^3 + 3x^2 - 4x - 12 = 0$ 55. $x^3 - 5x^2 + 3x + 9 = 0$
 56. $4x^3 - 16x^2 + 12x = 0$ 57. $2x^4 - 14x^3 + 12x^2 = 0$
 58. $4x^4 - 2x^2 - 4 = 2$ 59. $9x^4 - 9x^2 + 2 = 20$

60. **Open-Ended** To solve a polynomial equation, you can use any combination of graphing, factoring, and the Quadratic Formula. Write and solve an equation to illustrate each method.

 For Exercises 61 and 62, write a polynomial function to describe each volume. Then graph your function to solve each problem.

 61. **Geometry** Suppose a 2-in. slice is cut from one face of the cheese block as shown. The remaining solid has a volume of 224 in.^3 . Find the dimensions of the original block.



 62. **Geometry** The width of a box is 2 m less than the length. The height is 1 m less than the length. The volume is 60 m^3 . Find the length of the box.

Reading Math

For help with reading and solving Exercise 61, see p. 327.

63. $1 + i\sqrt{3}, 1 - i\sqrt{3}$ 64. $\frac{1}{2} + \frac{3i\sqrt{3}}{2}, \frac{1}{2} - \frac{3i\sqrt{3}}{2}$

65. $-\frac{3}{4} + \frac{\sqrt{73}}{4}, -\frac{3}{4} - \frac{\sqrt{73}}{4}$ 66. $-\frac{3}{4}, 1$

67. $y = (x + \frac{3}{2})^2 - \frac{13}{4}; (-\frac{3}{2}, -\frac{13}{4})$

68. $y = 2(x - \frac{1}{4})^2 - \frac{9}{8}; (\frac{1}{4}, -\frac{9}{8})$

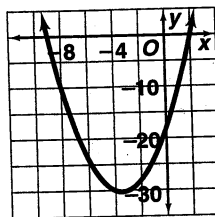
69. $y = (x + \frac{1}{2})^2 + \frac{7}{4}; (-\frac{1}{2}, -\frac{7}{4})$ 70. 2 real

solutions; $3 + \sqrt{7}, 3 - \sqrt{7}$ 71. 2 imaginary

solutions; $\frac{7}{4} + \frac{i\sqrt{31}}{4}, \frac{7}{4} - \frac{i\sqrt{31}}{4}$ 72. 2 real

solutions; $3 + \sqrt{5}, 3 - \sqrt{5}$

17. 2, -9



21. $y = x^3 - 18x^2 + 107x - 210$

23. $y = x^3 + 9x^2 + 15x - 25$ 29. -3 (mult. 3)

31. -1, 0, $\frac{1}{2}$ 37. 2 x^3 blocks, 15 x^2 blocks,

31 x blocks, 12 unit blocks 39. $V = 12x^3 - 27x$

41. $y = -2x^3 + 9x^2 - x - 12$

43. $y = 3x(x - 8)(x - 1)$ 45. $y = x^2(x + 4)(x - 1)$

47. 10.5, -7.1; $\frac{3}{2}, 4, 6$ 49. 2.98, -6.17; 1.5

51. Answers may vary. Sample:

$y = x^3 - 3x^2 - 10x$ 53. $y = x^4 - 4x^3 - 7x^2 +$

$22x + 24$ 55. 0 (mult. 2), -1 (mult. 2)

71. $-7x^4 - x^3$; quartic binomial 73. $(x + 4)(x + 1)$

75. $(x - 6)^2$ 77. -11

Chapter 6

Lesson 6-1

pp. 303-305

EXERCISES 1. $10x + 5$; linear binomial

3. $2m^2 + 7m - 3$; quadratic trinomial 13. $y = x^3 +$

1 15. $y = 1.5x^3 + x^2 - 2x + 1$ 19. $y = x^3 -$

$10x^2$; 2023 21. $y = -0.03948x^3 + 2.069x^2 -$

$17.93x + 106.9$; 206 25. $x^3 + 4x$; cubic binomial

27. 7; constant monomial 33. $-c^2 + 16$; binomial

35. $16x^2 - x - 5$; trinomial 47. $30x^3 - 10x^2$;

binomial 49. $b^3 - 6b^2 + 9b$; trinomial 67. 2

69.  5, -3)

71. $\begin{bmatrix} 2 & -3 & -6 \\ 5 & -2 & 0 \end{bmatrix}$

Extension

p. 306

1. (f, f) 2. (f, f) 3. (f, \) 4. (f, f) 5. (f, f)
6. (f, f) 7. (f, \) 8. (f, f) 9. (f, \) 10. (f, \)

Lesson 6-2

pp. 311-313

EXERCISES 1. $x^2 + x - 6$ 3. $x^3 - 7x^2 + 15x - 9$

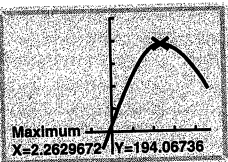
7. $x(x - 6)(x + 6)$ 9. $5x(2x^2 - 2x + 3)$

13. 24.2, -1.4; 0, -5, 1 15a. $h = x$,

$\ell = 16 - 2x$, $w = 12 - 2x$

b. $V = x(16 - 2x)(12 - 2x)$

c.  194 in.³, 2.26 in.



Lesson 6-3

pp. 318-320

EXERCISES 1. $x - 8$ 3. $x^2 + 4x + 3$, R 5 9. no

11. yes 13. $x^2 + 4x + 3$ 15. $x^2 - 11x + 37$,

R -128 23. $y = (x + 1)(x + 3)(x - 2)$ 25. $\ell = x +$

3 and $h = x$ 27. 0 29. 0 37. $x^2 + 4x + 5$

39. $x^4 - x^3 + x^2 - x + 1$ 43. yes 45. no

49. yes 51. no 53. $x^3 - 2x^2 - 2x + 4$, R -35

55. $x^3 - 4x^2 + x$ 65. $y = x^2 + 2x - 15$

67. $y = x^3 - 6x^2 + 3x + 10$ 69. 24 71. 23 - 11i

73. none exists

Lesson 6-4

pp. 324-326

EXERCISES 1. -2, 1, 5 3. 0, 1

13. $(x - 10)(x^2 + 10x + 100)$ 15. 3, $\frac{-3 \pm 3i\sqrt{3}}{2}$

17. 5, $\frac{-5 \pm 5i\sqrt{3}}{2}$ 21. $(x^2 - 7)(x - 1)(x + 1)$

23. $(x^2 - 3)(x - 2)(x + 2)$ 27. $\pm 3, \pm 1$ 29. $\pm 4, \pm 2i$

33. -1, 3.24, -1.24 35. -2, -3, 1 2 41. about

3.58 cm, about 2.83 cm 43. $\frac{4}{3}, \frac{-2 \pm 2i\sqrt{3}}{3}$

45. $\pm 5, \pm i\sqrt{2}$ 61. $V = x^2(4x - 2)$, 4 in. by 4 in.

by 16 in. 63. $-\frac{5}{2}, 1$; $y = (2x + 5)(x - 1)$

65. -1, 2, 2; $y = (x + 1)(x - 2)^2$

77. $2x^2 + x - 3$, R 2 79. ± 6 81. $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$

Answers

Extension

p. 328

1. pos. real roots: 1, neg. real roots: 2 or none
3. pos. real roots: 1, neg. real roots: 2 or none