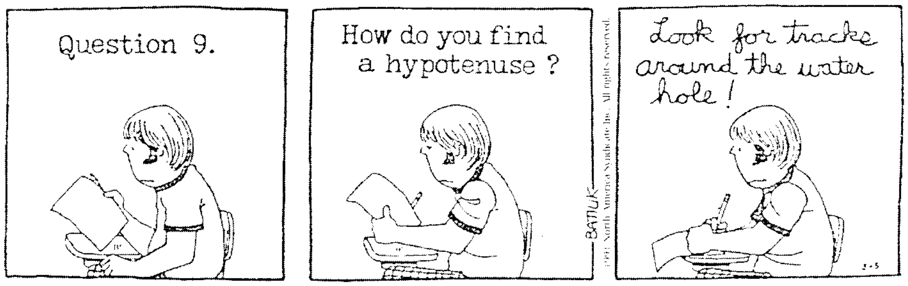
Lesson 10.1

The Theorem of Pythagoras



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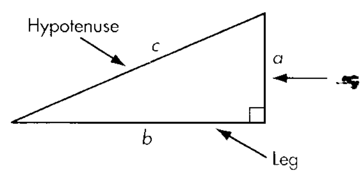
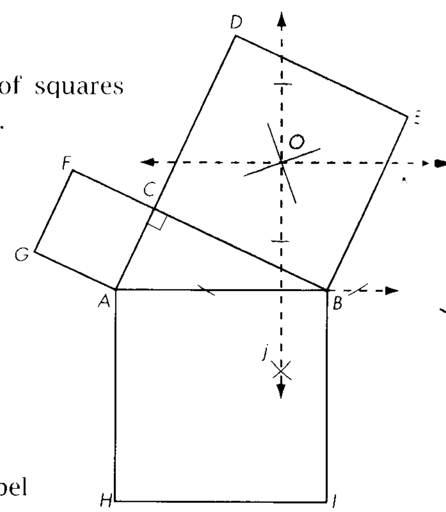
There is a surprising relationship between the lengths of the three sides of any right triangle. This property of right triangles is probably the most useful in all high school mathematics because it helps you calculate the distance between two points. Nobody knows at what point in history this relationship was first discovered. The ancient Babylonians and Chinese recognized this relationship, and some math historians believe that the ancient Egyptians also used a special case of this property of right triangles.

In a right triangle, the side opposite the right angle is called the **hypotenuse**. The other two sides are called **legs**. In the figure at right, a and b represent the lengths of the legs, and c represents the length of the hypotenuse. (And no, a hypotenuse is not a large animal that hangs out around watering holes.)



Investigation 10.1



There is a special relationship among the areas of squares constructed on the three sides of a right triangle. The dissection puzzle in this investigation is intended to get you thinking about this special relationship. On a full sheet of paper, perform Steps 1–6. Note that the arcs and the segment extensions necessary to complete Steps 1 and 2 are not indicated in the figure.

Step 1 Construct a scalene right triangle in the middle of your paper (hypotenuse down). Label it so that the hypotenuse is $\overline{AB}$ and the longer leg is $\overline{BC}$.

Step 2 Construct a square on each side of the triangle. Label the square on the longer leg *BCDE*. Label the square on the shorter leg *AGFC*. Label the square on the hypotenuse *ABIH*.

Step 3 Locate the center of *BCDE* (intersection of the two diagonals). Label the point *O*.

Step 4 Through point O, construct line j perpendicular to the hypotenuse.

Step 5 Through point O, construct line k perpendicular to line j. Line k is parallel to the hypotenuse. Lines j and k divide $BCDE$ into four parts.

Step 6 Cut out square $AGFC$ and the four parts of square $BCDE$. Arrange them to exactly cover square $ABIH$ on the hypotenuse.

If you are successful, then you have demonstrated that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two legs of your triangle. Compare your results with the results of others near you. If the lengths of the two legs of a right triangle are a and b, then the areas of the squares on the legs are a^2 and b^2. If the length of the hypotenuse is c, then the area of the square on the hypotenuse is c^2. State your observations as your next conjecture.

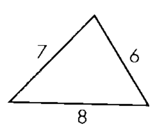
C-88 In a right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then --?-- (***Pythagorean Theorem***).

$a^2 + b^2 = c^2$

Your conjecture is known as the Pythagorean Theorem, named after Pythagoras (572–497 B.C.), the Greek philosopher who demonstrated that it is true. Recall that a theorem is a statement that has been proved. While you have discovered the relationship between the lengths of the sides of a right triangle, you have not actually proved it. However, we will call Conjecture 88 the Pythagorean Theorem because it is well known by that name.

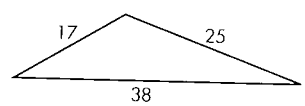
If the Pythagorean Theorem works for right triangles, does it work for all triangles? A quick check demonstrates that it doesn't hold for other triangles.

Acute triangle



$6^2 + 7^2 > 8^2$

Obtuse triangle

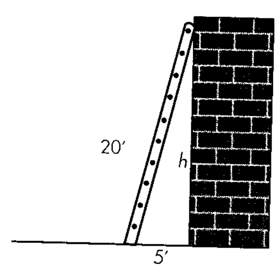


$17^2 + 25^2 < 38^2$

Let's look at a few examples to see how you can use the Pythagorean Theorem to find the distance between two points.

Example A

How high up on the wall will a twenty-foot ladder reach if the foot of the ladder is placed five feet from the wall?



$$(20)^2 = (5)^2 + (h)^2$$
$$400 = 25 + h^2$$
$$375 = h^2$$
$$\therefore\ h = \sqrt{375} \approx 19.4$$

Therefore the top of the ladder will touch the wall about 19.4 feet up from the ground.